CONSTRUCTION OF MIXED SAMPLING PLANS INDEXED THROUGH SIX SIGMA QUALITY LEVELS WITH TNT-(n₁, n₂; c) PLAN AS ATTRIBUTE PLAN

R. Radhakrishnan¹ and J. Glorypersial²

¹Associate Professor in Statistics, P.S.G College of Arts and Science, Coimbatore-641 014, Tamil Nadu, India.
²Assistant Professor in Statistics, Dr. G.R.D College of Science, Coimbatore-641 014, Tamil Nadu, India.

ABSTRACT
Six Sigma is a concept, a process, a measurement, a tool, a quality philosophy, a culture and a management strategy for the improvement in the system of an organization, in order to reduce wastages and increase the profit to the management and satisfaction to the customers. Six Sigma is a business improvement approach and management philosophy that seeks to find and remove causes of defects/errors in management processes by focusing on customer requirements, processes, and outputs that are of critical importance to constitute Six Sigma to be “a program for the near-elimination of defects from every product, process and transaction. Six Sigma is a strategic weapon that works across all processes, products, company functions and industries. Motorola [1] first adopted the concept of six sigma in their organization and established that it can produce less than 3.4 defects per million opportunities. Focusing on reduction of defects will result in more profit to the producer and enhanced satisfaction for the consumer. The concept of Six Sigma can be applied in the process of quality control in general and Acceptance sampling in particular. In this paper a procedure for the construction and selection of Mixed Sampling Plan indexed through Six Sigma Quality level having Tightened – Normal – Tightened plan of the type TNT-(n₁, n₂; c) plan as attribute plan is presented. The plans are constructed using SSQL-1 and SSQL-2 as indexing parameters. Tables are constructed for easy selection of the plan.

KEYWORDS: Six Sigma Quality Levels, Operating Characteristic Curve, Mixed Sampling Plan, Tightened – Normal – Tightened plan

I. INTRODUCTION
Mixed sampling plan is a two stage sampling procedure involving variables inspection in the first stage and attributes inspection in the second stage if the variables inspection of the first sample does not lead to acceptance. Mixed sampling plans are of two types, namely independent and dependent plans. Independent mixed sampling plans do not incorporate first sample results in the assessment of the second sample. Dependent mixed plans combine the results of the first and second samples in making a decision if a second sample is necessary. The mixed sampling has been designed under two cases of significant interest. In the first case the sample size n₁ is fixed and a point on the OC curve is given. In the second case plans are designed when two points on the OC curve are given. The mixed sampling plans are initially introduced by Dodge [2] and later developed by Bowker and Goode [3]. Schilling [4] has given a method for determining the operating characteristics for mixed variables-attributes sampling plans. Tightened-Normal-Tightened (TNT) sampling scheme was first

This paper deals with the construction of mixed variables – attributes sampling plan (independent case) using TNT-(n1, n2; c) plan as attribute plan indexed through Six Sigma Quality levels. Tables are constructed for easy selection of the plan and illustrations are also provided.

II. GLOSSARY OF SYMBOLS

The symbols used in this paper are as follows:

\( P \) : submitted quality of lot or process
\( P_a(p) \) : probability of acceptance for given quality \( p \)
\( P_1 \) : Probability of acceptance under tightened inspection
\( P_2 \) : Probability of acceptance under normal inspection
\( n_{1,1} \) : sample size for variable sampling plan
\( n_{1,2} \) : tightened (larger) sample size for attribute sampling plan
\( n_{2,2} \) : normal (smaller) sample size for attribute sampling plan
\( s \) : criterion for switching to tightened inspection
\( t \) : criterion for switching to normal inspection
\( \beta_j \) : probability of acceptance for lot quality \( p_j \)
\( \beta^p_j \) : probability of acceptance assigned to first stage for percent defective \( p_j \)
\( \beta^n_j \) : probability of acceptance assigned to second stage for percent defective \( p_j \)
\( k \) : variable factor such that a lot is accepted if \( \bar{X} \leq A = U - k \sigma \)

III. OPERATING PROCEDURE OF MIXED SAMPLING PLAN WITH TNT - (N1, N2; C) PLAN AS ATTRIBUTE PLAN

In this paper only independent mixed sampling plans are considered. The development of mixed sampling plans and the subsequent discussions are limited only to the upper specification limit \( U \). By symmetry a parallel discussion can be made use for lower specification limits. Also it is suggested that the mixed sampling plan with TNT-(n1, n2; 0) in the case of single sided specification \( (U) \), S.D \( (\sigma) \) known can be formulated by the parameters \( n_{1,2}, n_{2,2}, \) and \( c \). By giving the values for the parameters an independent plan for single sided specification, \( \sigma \) known would be carried out as follows:

1. Determine the parameters with reference to ASN and OC curves
2. Take a random sample of size \( n_{1,1} \) from the lot assumed to be large
3. If a sample average \( \bar{X} \leq A = U - k \sigma \), accept the lot
   If the sample average \( \bar{X} > A = U - k \sigma \), take another sample of size \( n_{1,2} \)
   (i) inspect using tightened inspection with a larger sample size \( n_{1,2} \) and \( c = 0 \).
   (ii) switch to normal inspection when ‘t’ lots in a row are accepted under tightened inspection.
   (iii) inspect using normal inspection with smaller sample size \( n_{2,2} \) and \( c=0 \).
   (iv) switch to tightened inspection after an additional lot is rejected in the next ‘s’ lots
When $\sigma$ is not known, simply substitute the sample standard deviation ($s_1$) where

$$s_1 = \sqrt{\frac{\sum_{i=1}^{n} (X_i - \overline{X})^2}{n-1}}$$

for $\sigma$ in the known standard deviation procedure by choosing an appropriate value of ‘$k$’ and sample size ‘n’ for the unknown standard deviation case.

IV. CONDITIONS FOR APPLICATIONS

(i) Production process should be steady and continuous
(ii) Lots are submitted sequentially in the order of their production
(iii) Inspection is by variable in the first stage and attribute in the second stage with quality defined as the fraction defective
(iv) Human involvement should be less in the process

V. DEFINITION OF SSQI-1 AND SSQI-2

The proportion defective corresponding to the probability of acceptance of the lot as $1-3.4 \times 10^{-6}$, (the concept of six sigma quality suggested by Motorola [1] in the OC curve is termed as Six Sigma Quality Level-1 (SSQL-1). This new sampling plan is constructed with a point on the OC curve (SSQL-1 $\beta_1$), where $\beta_1 = 1-\alpha_1$ and $\alpha_1 = 3.4 \times 10^{-6}$ suggested by Radhakrishnan and Sivakumaran [12]. Further the proportion defective corresponding to the probability $2\alpha_1$ in the OC curve is termed as Six Sigma Quality Level-2 (SSQL-2). This new sampling plan is constructed with a point on the OC curve (SSQL-2 $\beta_2$), where $\beta_2=2\alpha_1$ suggested by Radhakrishnan and Sivakumaran [12].

VI. DESIGNING THE MIXED SAMPLING PLAN WHEN A SINGLE POINT ON THE OC CURVE IS KNOWN

The procedure for the construction of mixed variables – attributes sampling plans is provided by Schilling [4] for a given ‘$n_1$’ and a point ‘$p_1$’ on the OC curve. A modified procedure for the construction of independent mixed variables – attributes sampling plan for a given SSQL-1, SSQL-2 and ‘$n_1$’ is given below.

♦ Split the probability of acceptance ($\beta_i$) determining the probability of acceptance that will be assigned to the first stage. Let it be $\beta_i'$.

♦ Decide the sample size $n_1$ (for variable sampling plan) to be used

♦ Calculate the acceptance limit for the variable sampling plan as

$$A = U - k \sigma = U - [z (p) + \{z (\beta_1') / \sqrt{n_1} \}] \sigma,$$

where $z (t)$ is the standard normal variate corresponding to ‘$t$’ such that

$$t = \int_{2}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-u^2/2} du$$

♦ Determine the sample average $\overline{X}$. If a sample average $\overline{X} > A = U - k \sigma$, take a second stage sample of size ‘$n_2$’ using attribute sampling plan.

♦ determine $\beta''$, the probability of acceptance assigned to the attributes plan associated with the second stage sample as $\beta'' = (\beta_i - \beta_i') / (1-\beta_i')$

♦ Determine the appropriate second stage sample of size ‘$n_{2,2}$’ from $Pa (p) = \beta''$ for $p= p_i$

♦ Now determine $\beta_1''$, the probability of acceptance assigned to the attributes plan associated with the second stage sample as $\beta_1'' = (\beta_i - \beta_i') / (1-\beta_i')$.

♦ Determine the appropriate second stage sample of size ‘$n_{2,2}$’ and ‘c’ from $Pa (p) = \beta_1''$ for $p=SSQL-1$.

♦ Determine $\beta_2''$, the probability of acceptance assigned to the attributes plan associated with the second stage sample as $\beta_2'' = (\beta_i - \beta_i') / (1-\beta_i')$. 
Determine the appropriate second stage sample of size $n_2$ and $c$ from $P_a(p) = \beta$ for $p=SSQL-2$.

Using the above procedure tables can be constructed to facilitate easy selection of mixed sampling plan with TNT-$(n_1, n_2; c)$ plan as attribute plan indexed through SSQL-1 and SSQL-2.

VII. OPERATING CHARACTERISTIC FUNCTION

Under the assumption of Poisson model, the OC function of the independent mixed sampling plan having TNT-$(n_1, n_2; c)$ Plan is given by

$$P_a(p) = P(\overline{X} \leq A) + [1 - P(\overline{X} \leq A)]$$

$$= \frac{P_1(1 - P'_1)(1 - P'_1)(1 - P'_1) + P_2 P'_1}{(1 - P'_1)(1 - P'_1)(1 - P'_1) + P'_1(1 - P'_1)(1 - P'_1)}$$

(1)

Where

$$P_1 = \sum_{t=0}^{c} \frac{e^{-n_1 p}(n_1 p)^t}{t!}$$

(2)

$$P_2 = \sum_{t=0}^{c} \frac{e^{-n_2 p}(n_2 p)^t}{t!}$$

(3)

$P_1$ = Probability of acceptance under tightened inspection

$P_2$ = Probability of acceptance under normal inspection

Since $n_{1,2} > n_{2,2}$, we set $n_{1,2}$ equal to some multiple of $n_{2,2}$ say, $kn_{2,2}$.

VIII. CONSTRUCTION OF MSP WITH TNT- $(N1, N2; C)$ PLAN AS ATTRIBUTE PLAN INDEXED THROUGH SSQL-1

In this section the mixed sampling plan indexed through SSQL-1 is constructed. A point on the OC curve can be fixed such that the probability of acceptance of fraction defective SSQL-1 is $\beta_1$. The general procedure given by Schilling [4] is used for constructing the mixed sampling plan as attribute plan indexed through SSQL-1 [for $\beta_1" = (\beta_1 - \beta_1') / (1-\beta_1')$] with $\beta_1=0.9999966$ and $\beta_1'=0.50$, the $n_{2,SSQL-1}$ values are calculated for different values of $c$ and $k$ using visual basic program and is presented in Table1. The sample size $n_{1,2} = n_{2,2}$ of the normal plan is obtained as $n_{2,2} = n_{2,SSQL-1}/SSQL-2$ and then the sample size $n_{1,2}$ of the tightened plan is found as $n_{1,2} = kn_{2,2}$ ($k > 1$). Hence the parameters of the TNT-$(n_1, n_2; c)$ schemes $n_{1,2}, n_{2,2}$ and $c$ are obtained for various values of SSQL-1.

The sigma level of the process [24] is calculated using the Process Sigma Calculator by providing the sample size and acceptance number.

8.1 Selection of the plan

Table 1 is used to construct the plans when SSQL-1, $s$ and $t$ are given. For any given values of SSQL-1, $c$ and $k$ one can determine $n_{2,2}$ value using $n_{2,2} = n_{2,SSQL-1}/SSQL-1$.

8.2 Example

Given SSQL-1=0.00000002, $c = 1$, $k = 2.0$ and $\beta_1'=0.50$, the value of SSQL-1 is selected from Table 1 as 0.00000008 and the corresponding sample size of normal plan $n_{2,2}$ is computed as $n_{2,2} = n_{2,SSQL-1}/SSQL-1=0.00000002$. The sample size of tightened plan $n_{1,2}$ is computed as $n_{1,2} = (2.0)(340) = 680$, which are associated with 4.3 and 4.5 sigma levels respectively. For a fixed $\beta_1'=0.50$, the Mixed Sampling Plan with TNT-$(n_1, n_2; c)$ Plan as attribute plan are $n_{1,2}=680, n_{2,2}=340$ and $c = 1$ for a specified SSQL-1=0.00000002.

8.3 Practical Application

Suppose the plan with $n_1 = 10, c=1$ and $k=2.0$ is to be applied to the lot-by-lot acceptance inspection of solar mobile phone charger. The characteristic to be inspected is the “weight of the solar mobile
phone charger in g” for which there is a specified upper limit (U) of 90 g with a known standard deviation (σ) of 0.002 g. In this example, U=90 g, σ = 0.002 g and k = 2.0.

Now, in applying the variable inspection first, take a random sample of size n₁=10 from the lot. Record the sample results and find \( \bar{X} \). If \( \bar{X} \leq A = U - k \sigma = 89.96 \) g, accept the lot otherwise take a random sample of size 680 and apply attribute inspection.

Under attribute inspection, the TNT-(\( n_1 \), \( n_2 \); c) plan as attribute plan, if the manufacturer of solar mobile phone charger fixes the quality of cell phones as SSQL-1 = 0.00000002 (2 non-conforming solar mobile phone chargers out of 10 crore items), then inspect under tightened inspection with sample of size 680 solar mobile phone chargers and acceptance number c = 1 from the manufactured lot of a particular month. If 5 lots in a row are accepted under tightened inspection, then switch to normal inspection. Then inspect under normal inspection with a sample of 340 solar mobile phone chargers and acceptance number c = 1 from the manufactured lot of a particular month. Switch to tightened inspection, after a rejection, if an additional lot is rejected in the next 4 lots and inform the management for corrective action. The OC curve of the plan in Example 8.2 is presented in the Figure 1.

**Table 1:** Various characteristics of the MSP when (SSQL-1, \( \beta_1 \)) is known with \( \beta_1 = 0.9999966, \beta_1' = 0.50 \).

<table>
<thead>
<tr>
<th>c</th>
<th>k</th>
<th>( n_2)SSQL-1</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.25</td>
<td>0.00000068</td>
</tr>
<tr>
<td>0</td>
<td>1.50</td>
<td>0.00000068</td>
</tr>
<tr>
<td>1</td>
<td>1.25</td>
<td>0.00000068</td>
</tr>
<tr>
<td>1</td>
<td>1.50</td>
<td>0.00000068</td>
</tr>
<tr>
<td>1</td>
<td>1.75</td>
<td>0.00000068</td>
</tr>
<tr>
<td>1</td>
<td>2.00</td>
<td>0.00000068</td>
</tr>
<tr>
<td>1</td>
<td>2.25</td>
<td>0.00000068</td>
</tr>
<tr>
<td>1</td>
<td>2.50</td>
<td>0.00000068</td>
</tr>
<tr>
<td>1</td>
<td>2.75</td>
<td>0.00000068</td>
</tr>
<tr>
<td>1</td>
<td>3.00</td>
<td>0.00000068</td>
</tr>
<tr>
<td>2</td>
<td>2.00</td>
<td>0.00000068</td>
</tr>
<tr>
<td>2</td>
<td>2.25</td>
<td>0.00000068</td>
</tr>
<tr>
<td>2</td>
<td>2.50</td>
<td>0.00000068</td>
</tr>
<tr>
<td>2</td>
<td>2.75</td>
<td>0.00000068</td>
</tr>
<tr>
<td>3</td>
<td>2.00</td>
<td>0.00000068</td>
</tr>
<tr>
<td>3</td>
<td>2.25</td>
<td>0.00000068</td>
</tr>
<tr>
<td>3</td>
<td>2.50</td>
<td>0.00000068</td>
</tr>
<tr>
<td>3</td>
<td>2.75</td>
<td>0.00000068</td>
</tr>
</tbody>
</table>
IX. CONSTRUCTION OF MSP WITH TNT- (n1, n2; c) PLAN AS ATTRIBUTE PLAN INDEXED THROUGH SSQL-2

In this section the mixed sampling plan indexed through SSQL-2 is constructed. A point on the OC curve can be fixed such that the probability of acceptance of fraction defective SSQL-2 is β2. The general procedure given by Schilling [4] is used for constructing the mixed sampling plan as attribute plan indexed through SSQL-2 [for β" = (β2 - β2') / (1-β2')] with β2 = 0.0000068 and β2' = 0.0000034, the n2 SSQl-2 values are calculated for different values of c and k using visual basic program and is presented in Table2. The sample size 'n1,2 = n2,2' of the normal plan is obtained as n2,2 = n2,2 SSQl-2/SSQl-2 and then the sample size 'n1,2' of the tightened plan is found as n1,2 = kn2,2 (k >1). Hence the parameters of the TNT-(n1, n2; c) schemes n1,2 , n2,2 and c are obtained for various values of SSQl-2.

9.1 Example

Given SSQl-2=0.008, c = 2, k = 2.0 and β1'=0.50, the value of SSQl-2 is selected from Table 2 as 7.4418576 and the corresponding sample size of normal plan n2,2 is computed as n2,2 = n2,2 SSQl-2/SSQl-2 = 7.4418576 /0.008 =930 and the sample size of tightened plan n1,2 is computed as n1,2 = (2.0)(930) = 1860, which are associated with 4.4 and 4.6 sigma levels respectively. For a fixed β1'=0.50, the Mixed Sampling Plan with TNT-(n1, n2; c) Plan as attribute plan are n1,2 =1860, n2,2 = 930 and c = 2 for a specified SSQl-2=0.008.

9.2 Practical Application

Suppose the plan with n1 = 10, c=2 and k=2.0 is to be applied to the lot-by-lot acceptance inspection of solar mobile phone charger. The characteristic to be inspected is the “weight of the handy mobile phone charger in g” for which there is a specified upper limit (U) of 104 g with a known standard deviation (σ) of 0.002 g. In this example, U=104 g, σ = 0.002 g and k = 2.0

Now, in applying the variable inspection first, take a random sample of size n1=10 from the lot. Record the sample results and find X̄. If X̄ ≤ A = U – kσ=103.96 g, accept the lot otherwise take a random sample of size 1860 and apply attribute inspection.

Under attribute inspection, the TNT-(n1, n2; c) plan as attribute plan, if the distributor of handy mobile phone charger fixes the quality of mobile phone chargers as SSQl-2 = 0.008 (8 non-conforming handy mobile phone chargers out of 1 thousand items), then inspect under tightened inspection with sample of size 1860 handy mobile phone chargers and acceptance number c = 2 from the manufactured lot of a particular month. If 5 lots in a row are accepted under tightened inspection, then switch to normal inspection. Then inspect under normal inspection with a sample of 930 handy mobile phone chargers and acceptance number c = 2 from the manufactured lot of a particular month. Switch to tightened inspection, after a rejection, if an additional lot is rejected in the next 4 lots and inform the management for corrective action. The OC curve of the plan in Example 9.1 is presented in the Figure 2.

<table>
<thead>
<tr>
<th>X</th>
<th>y</th>
<th>β</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>3.00</td>
<td>0.0000068</td>
</tr>
<tr>
<td>4</td>
<td>2.50</td>
<td>0.0000068</td>
</tr>
<tr>
<td>4</td>
<td>2.75</td>
<td>0.0000068</td>
</tr>
<tr>
<td>4</td>
<td>3.00</td>
<td>0.0000068</td>
</tr>
<tr>
<td>5</td>
<td>2.50</td>
<td>0.0000068</td>
</tr>
<tr>
<td>5</td>
<td>2.75</td>
<td>0.0000068</td>
</tr>
<tr>
<td>5</td>
<td>3.00</td>
<td>0.0000068</td>
</tr>
</tbody>
</table>
X. Conclusion

This paper provides a procedure to engineers for the selection of Mixed Sampling Plan through Six Sigma Quality Levels having TNT-\((n_1, n_2; c)\) Plan as attribute plan. These plans are very effective in
place of classical plans indexed through SSQL-1 and SSQL-2 and these plans are useful for the companies in developed and developing countries which are practicing six sigma quality initiatives in their process. The use of SSQL-1 and SSQL-2 schemes results in savings in the companies reduce costs related to scrap, rework, inspection, and customer dissatisfaction, when compared with single-sampling plan. These schemes are suitable when we have a without break stream of batches or lots, where quality shifts slowly and when the submitted lots are expected to be of in essence the same quality. The procedure outlined in this paper can be used for other plans also.

REFERENCES


**Authors**

**R. Radhakrishnan** has 31 years of experience in teaching, published 75 papers in national and international journals, presented more than 150 papers in national and international conferences, guided 30 MPhil’s and six PhD scholars and acquired Six Sigma Black Belt certification. He is an ISO 9001 Auditor and a Six Sigma Consultant. In addition to research degrees such as MPhil and PhD, he has also acquired his Masters in Business Administration. He is the Reviewer for ten international journals and Associate Editor for two journals.

**J. Glorypersial** has eight years of experience in teaching and published papers in international journals and presented papers in national conferences. She is working under the guidance of Dr. R. Radhakrishnan for her PhD in the field of quality control.