

# ROBUSTNESS OF FEEDBACK FIRST-ORDER LAG-LEAD, FEEDFORWARD SECOND-ORDER LAG-LEAD AND FEEDFORWARD FIRST-ORDER LAG-LEAD COMPENSATORS USED WITH SECOND-ORDER PROCESSES

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## ABSTRACT

*Robustness is one of the requirements used in controllers and compensators design. The designs presented in the previous papers did not consider the robustness of the controller or compensator. Therefore, the objective of this paper is to investigate the robustness of feedback first-order lag-lead, feedforward second-order lag-lead and feedforward first-order lag-lead compensators used to control second-order processes against uncertainty in the process parameters. A variation of  $\pm 20\%$  in process parameters is considered through simulation to study its effect on the system performance parameters using the tuned controllers. With a feedback first-order lag-lead compensator controlling a highly oscillating second order process, the variation in process natural frequency and damping ratio has small effect on the maximum percentage overshoot, settling time and the phase margin of the control system. The phase margin is above 74 degrees for all the changes in the process parameters providing robust compensator characteristics. The phase margin of the control system using the proposed compensators for the range of the variation of the process parameters is above 56 degrees, the maximum percentage overshoot for a step input is less than 11.5 % and the settling time does not exceed 1.41 seconds indicating the robustness of the three compensators.*

**KEYWORDS:** *Feedback First-order Lag-lead Compensator, Feedforward First-order and Second-order Lag-lead Compensators, Compensators Robustness, Control System Performance.*

## I. INTRODUCTION

Processes are subject to uncertainty in their parameters during operation. Therefore, it is worth to investigate the effectiveness of the used compensators with such uncertainty. Hu, Chang, Yeh and Kwatny (2000) used the  $H_\infty$  approximate I/O linearization formulation and  $\mu$ -synthesis to design a nonlinear controller for an aircraft longitudinal flight control problem and address tracking, regulation and robustness issues [1]. Gong and Yao (2001) generalized a neural network adaptive robust control design to synthesize performance oriented control laws for a class of nonlinear systems in semi-strict feedback forms through the incorporation of backstepping design techniques [2]. Lee and Na (2002) designed a robust controller for a nuclear power control system. They used the Kharitonov and edge theorem in the determination of the controller which was simpler than that obtained by the  $H_\infty$  [3]. Arvanitis, Syrkos, Stellas and Sigrimis (2003) analyzed PDF controllers designed and tuned to control integrator plus dead time processes in terms of robustness. They performed the robustness analysis in terms of structured parametric uncertainty description [4]. Lhommeau, Hardouin, Cottenceau and Laulin (2004) discussed the existence and the computation of a robust controller set for uncertain systems described by parametric models with unknown parameters assumed to vary between known bounds [5]. Dechanupaprittha, Hongesombut, Watanabe, Mitani and Ngammroo (2005) proposed the design of robust superconducting magnetic energy storage controller in a multimachine power system by using hybrid tabu search and evolutionary programming. The objective function of the

optimization problem considered the disturbance attenuation performance and robust stability index [6].

Chin, Lau, Low and Seet (2006) proposed a robust PID controller based on actuated dynamics and an unactuated dynamics shown to be global bounded by the Sordalen lemma giving the necessary sufficient condition to guarantee the global asymptotic stability of the URV system [7]. Vagja and Tzes (2007) designed a robust PID controller coupled into a Feedforward compensator for set point regulation of an electrostatic micromechanical actuator. They tuned the PID controller using the LMI-approach for robustness against the switching nature of the linearized system dynamics [8]. Fiorentini and Bolender (2008) described the design of a nonlinear robust/adaptive controller for an air-breathing hypersonic vehicle model. They adapted a nonlinear sequential loop-closure approach to design a dynamic state-feedback control for stable tracking of velocity and altitude reference trajectories [9]. Labibi, Marquez and Chen (2009) presented a scheme to design decentralized robust PI controllers for uncertain LTI multi-variable systems. They obtained sufficient conditions for closed-loop stability of multi-variable systems and robust performance of the overall system [10]. Matusu, Vanekova, Porkop and Bakosova (2010) presented a possible approach to design simple PI robust controllers and demonstrate their applicability during control of a laboratory model with uncertain parameters through PLC [11].

Kada and Ghazzawi (2011) described the structures and design of a robust PID controller for higher order systems. They presented a design scheme combining deadbeat response, robust control and model reduction techniques to enhance the performance and robustness of the PID controller [12]. Surjan (2012) applied the genetic algorithm for the design of the structure specified optimal robust controllers. The parameters of the chosen controller were obtained by solving the nonlinear constrained optimization problem using IAE, ISE, ITAE and ITSE performance indices. He used constraints on the frequency domain performances with robust stability and disturbance rejection [13]. Jiao, Jin and Wang (2013) analyzed the robustness of a double PID controller for a missile system by changing the aerodynamic coefficients. They viewed the dynamic characteristics as a two-loop system and designed an adaptive PID control strategy for the pitch channel linear model of supersonic missile [14]. Hassaan (2014) studied the robustness of a number of controllers and compensators from the second generation of PID-based controllers and lag-lead compensators [15-17].

## II. ANALYSIS

The process considered in this analysis has the transfer function,  $G_p(s)$ :

$$G_p(s) = \omega_{np}^2 / (s^2 + 2\zeta_p\omega_{np}s + \omega_{np}^2) \quad (1)$$

Where:

$$\begin{aligned} \omega_{np} &= \text{process natural frequency} = 10 \quad \text{rad/s.} \\ \zeta_p &= \text{process damping ratio} = 0.05 \end{aligned}$$

### 2.1. Feedback First-order Lag-lead Compensator Tuning

Hassaan suggested this compensator to control this highly oscillating second-order process [18]. The compensator has 4 parameters:

- Proportional gain of the feedforward proportional module:  $K_{pc}$ .
- The compensator gain:  $K_c$ .
- The compensator zero time constant:  $T_z$ .
- The compensator pole time constant:  $T_p$ .

The tuning parameters and the control system performance measures are:

$$\begin{aligned} K_{pc} &= 0.9579 \\ K_c &= 0.00872 \\ OS_{max} &= 0.0993\% \\ T_s &= 0.3886 \text{ s} \\ GM &= \infty \quad \text{dB} \\ PM &= 87.3 \quad \text{degrees} \end{aligned}$$

### 2.2. Process Uncertainty

Due to the change in the operating conditions during operation, the process is subjected to parametric changes. It is assumed that this change can be as large as  $\pm 20\%$  of the assigned process parameters.

### 2.3. Feedback First-order Lag-lead Compensator Robustness

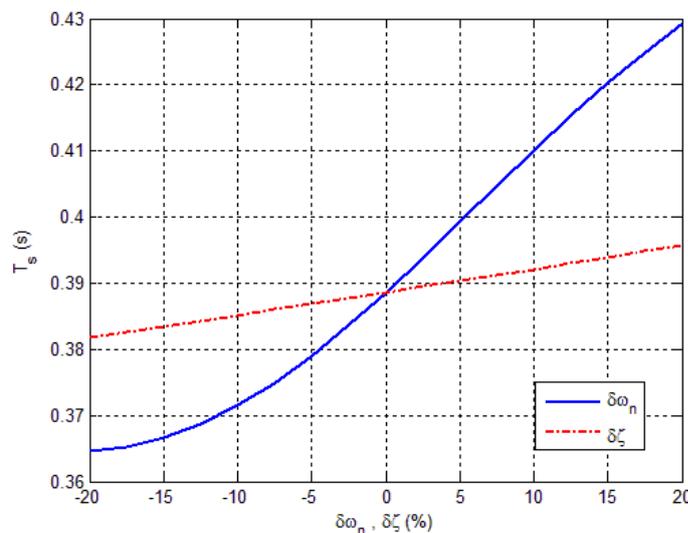
The control system is robust when it has acceptable changes in its performance due to model changes or inaccuracy [19]. On the other hand Lee and Na added the stability requirement to the robustness definition besides the plants having uncertainty [3]. Toscano added that the controller has to be able to stabilize the control system for all the operating conditions [20].

In this work, the robustness of the controller and hence of the whole control system is assessed as follows:

- A nominal process parameters are identified.
- The compensator is tuned for those process parameters.
- A variation of the process parameters is assumed within a certain range.
- Using the same compensator parameters, the step response of the system using the new process parameters is drawn and the control system performance is evaluated through the maximum percentage overshoot and settling time.
- The frequency based relative stability parameters are also evaluated using the open-loop transfer function of the control system.
- The variation in process parameters is increased and the procedure is repeated.

Application of the above procedure results in the fact that with the feedback first-order lag-lead compensator almost all the performance parameters change with changing the process parameters but within accepted limits.

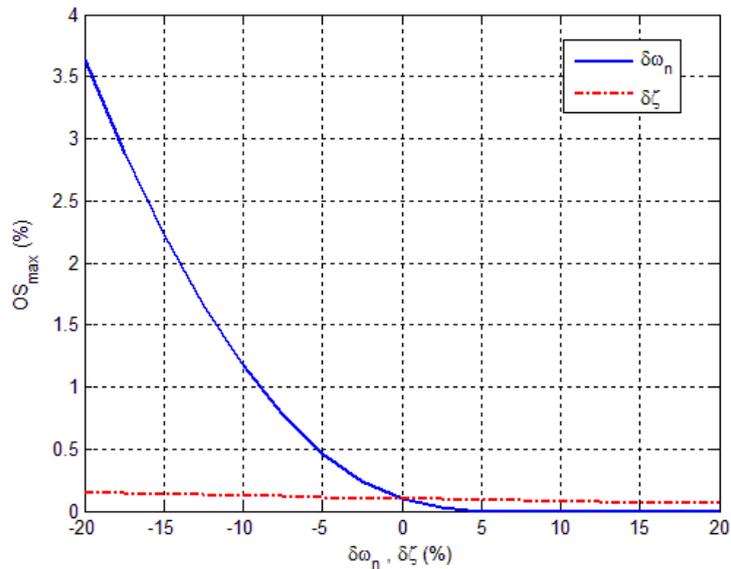
- The gain margin did not change from its infinity limit.
- Fig.1 shows the variation of the settling time against the variation in the process parameters.



**Figure 1.** Effect of process parameters change on system settling time

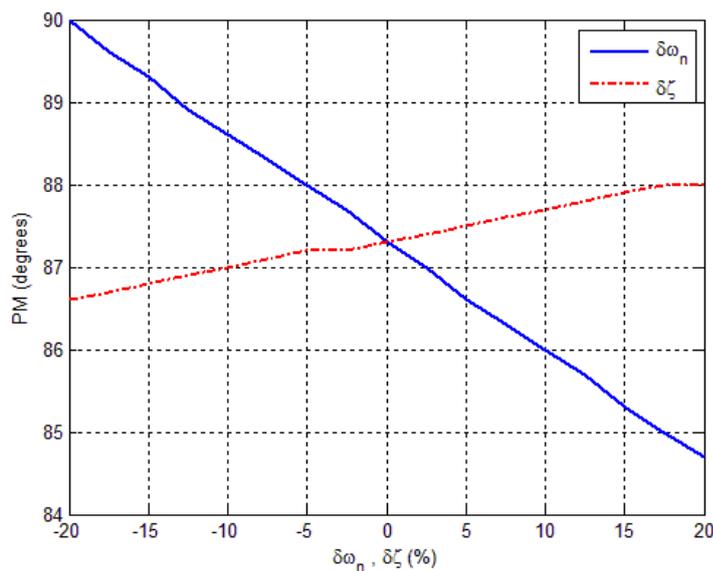
(Feedback first-order lag-lead compensator).

- Fig.2 shows the variation of the maximum percentage overshoot against the variation in the process parameters.



**Figure.2.** Effect of process parameters change on system maximum overshoot  
 (Feedback first-order lag-lead compensator).

- Fig.3 shows the variation of the phase margin against the variation in the process parameters.



**Figure 3.** Effect of process parameters change on system phase margin  
 (Feedback first-order lag-lead compensator).

#### 2.4.Feedforward Second-order Compensator Controlling a Highly Oscillating Second Order Process

Hassaan used a tuning approach based on an ITAE objective function to tune a feedforward second-order compensator when used with a highly oscillating second-order process [21].

The compensator parameters and the system performance measures are:

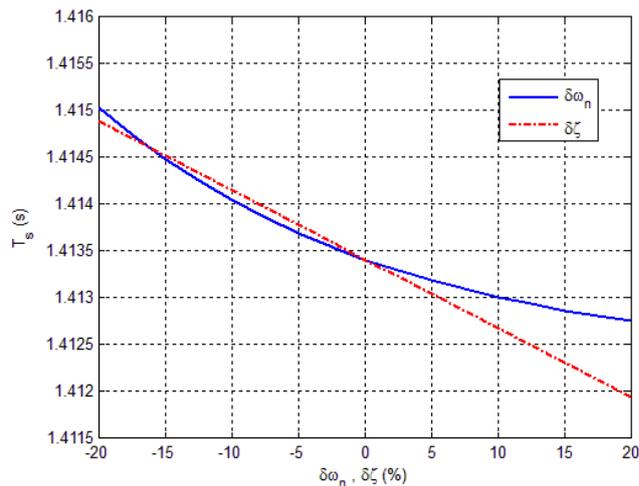
$$\begin{aligned} K_c &= 100 \\ T_1 &= 0.332706 \\ T_2 &= 0.3948461 \\ b &= 48.575667 \end{aligned}$$

Maximum percentage overshoot:	7.994	%
Settling time:	1.379	s
Gain margin:	$\infty$	

Phase margin: 62.1 degrees

The robustness investigation procedure is applied on the resulting control system for process variation in the range  $\pm 20\%$  from the nominal values. The results are as follows:

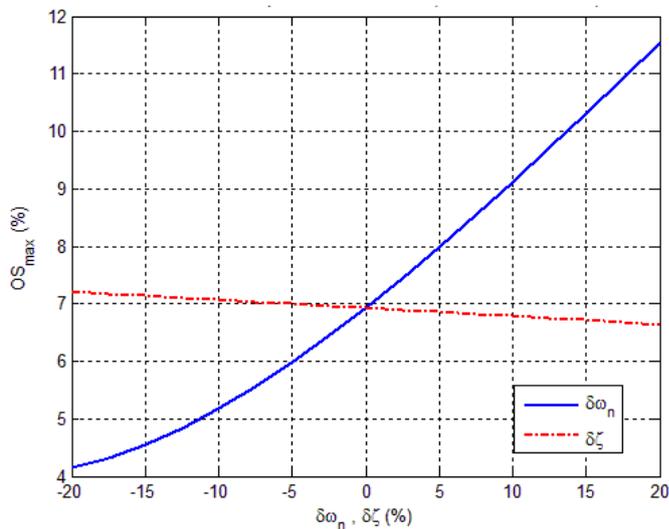
- The gain margin of the control system is infinity for all the changes in process parameters.
- The change in settling time is negligible (0.12 % maximum).
- The maximum change in the maximum percentage overshoot with natural frequency and damping ratio change is about 66.5 % and 4 % respectively.
- The maximum change in the phase margin with natural frequency and damping ratio change is 9.2 % and 0.32 % respectively.
- Fig.4 shows the effect of the natural frequency change on the system settling time.



**Figure 4.** Effect of process parameters change of system settling time.

(feedforward second-order compensator).

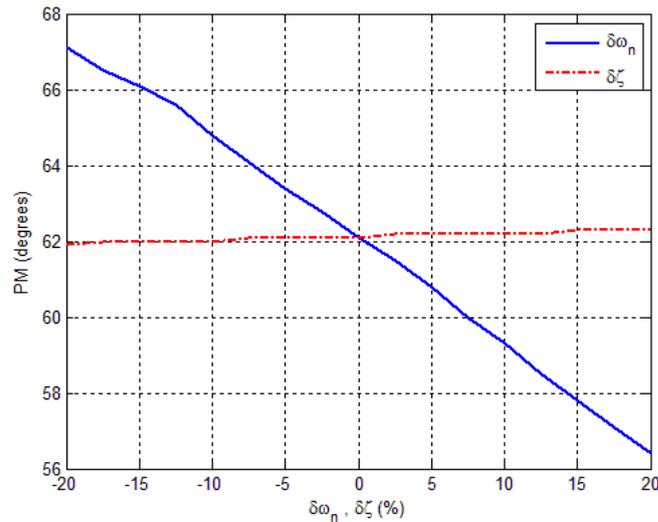
- Fig.5 shows the variation of the maximum percentage overshoot against the variation in the process parameters.



**Figure 5.** Effect of process parameters change on system maximum overshoot.

(feedforward second-order compensator).

- Fig.6 shows the variation of the phase margin against the variation in the process parameters.



**Fig.6** Effect of process parameters change on system phase margin (feedforward second-order compensator).

### 2.5. Feedforward First-order Compensator Controlling a Very Slow Second Order Process

Hassaan presented a tuning approach to tune a feedforward first-order compensator when used with a very slow second order process using an ISE objective function [22].

The compensator parameters and the system performance measures are:

$$K_c = 80.063$$

$$T_z = 30.1813$$

$$T_p = 14.6565$$

Maximum percentage overshoot = 0

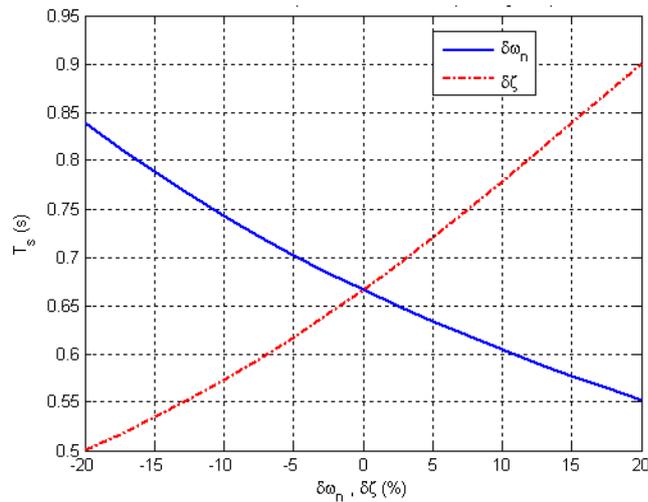
Settling time = 0.666 s

Gain margin = infinity

Phase margin = 73.5 degrees

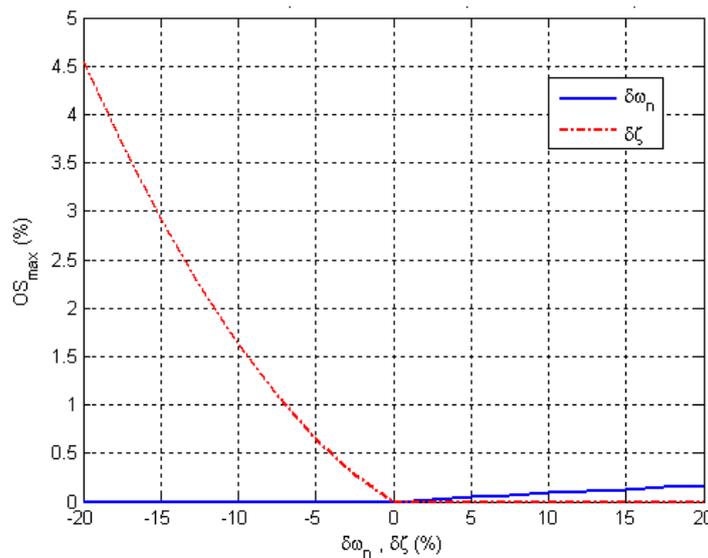
The robustness investigation procedure is applied on the resulting control system for process variation in the range  $\pm 20\%$  from the nominal values. The results are as follows:

- The maximum change in settling time is less than 26 % for the maximum change of 20 % in the process natural frequency.
- The maximum percentage overshoot is not affected by the negative changes in process natural frequency.
- The maximum percentage overshoot does not exceed 0.17 % for the maximum change of 20 % in the process natural frequency.
- The maximum decrease in phase margin does not exceed 0.28 % for the maximum change of -20 % in the process natural frequency.
- The maximum percentage overshoot is not affected by the positive changes in process damping ratio.
- The maximum increase in settling time is less than 35 % for the maximum change of 20 % in the process damping ratio.
- The maximum decrease in phase margin does not exceed 10.5 % for the maximum change of -20 % in the process damping ratio.
- Fig.7 shows the effect of the natural frequency and damping ratio change on the system settling time.



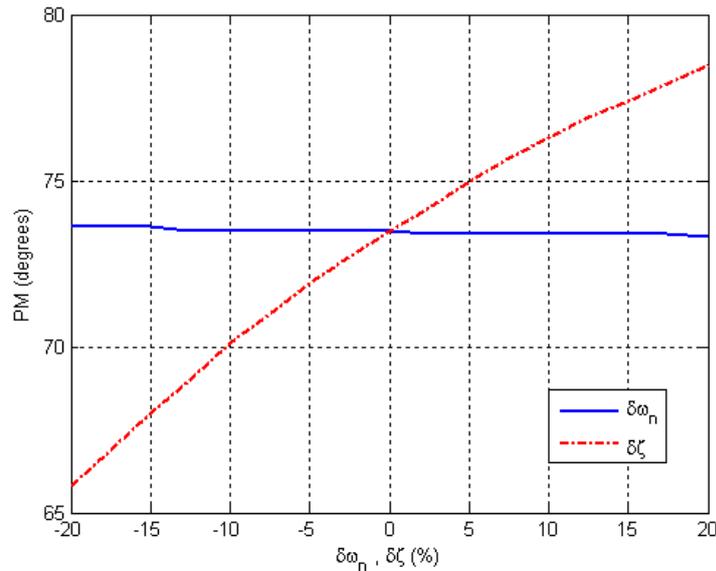
**Figure 7.** Effect of process parameters change on system settling time (feedforward first-order compensator).

- Fig.8 shows the effect of the natural frequency and damping ratio change on the system maximum percentage overshoot.



**Figure 8.** Effect of process parameters change on system maximum percentage overshoot (feedforward first-order compensator).

- Fig.9 shows the effect of the natural frequency and damping ratio change on the system phase margin.



**Figure 9.** Effect of process parameters change on system phase margin (feedforward first-order compensator).

### III. CONCLUSIONS

- Variation in second-order process parameters within  $\pm 20\%$  was considered.
- The judgment on the robustness condition of a controller is based on an accepted range of both gain margin and phase margin of the closed-loop control system.
- According to Ogata [23], a recommended range is:  $GM \geq 6$  dB and  $30 \leq PM \leq 60$  degrees.
- According to Lei and Man [24], the phase margin range can be widened to be:  $30 \leq PM \leq 90$  degrees.
- Tuned feedback first-order lag-lead, feedforward second-order lag-lead and feedforward first-order lag-lead compensators used to control a second order process are robust since they controlled the process for set-point change maintaining acceptable performance for the closed-loop control system for the range of parameters change.
- For the  $\pm 20\%$  change in process parameters, the minimum phase margin was 56.4 degrees for the 3 studied compensators indicating the acceptable performance of the control system.
- The maximum percentage overshoot corresponding to the minimum phase angle was 11.5%.
- The settling time in the 3 cases did not exceed 1.41 seconds indicating the fast response of the control system.
- The maximum percentage overshoot of the control system using the feedback first-order compensator and the feedforward second-order compensator was more sensitive to the variation in the process natural frequency.
- The settling time of the control system using the feedforward first-order compensator was more sensitive to the variation in both the process natural frequency and damping ratio.
- The three compensators studied in this paper are robust since they generate feedback control systems having acceptable performance over the variation range in the process parameters ( $\pm 20\%$ ).

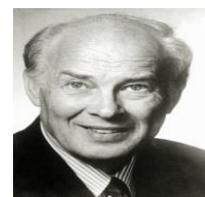
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## DEDICATION

I dedicate this work to my Ph.D. supervisor Prof. John Parnaby who was The chairman of the Manufacturing Systems Engineering Department of Bradford University during the 1970's. Prof. Parnaby died on 5<sup>th</sup> January 2011 in UK.



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- Emeritus professor of System Dynamics and Automatic Control.
- He has got his Ph.D. from Bradford University, UK in 1979 under the Supervision of Prof. John Parnaby.
- Research on Automatic Control, Mechanical Vibrations, Mechanism Synthesis and History of Mechanical Engineering.
- Served in a large number of universities in Africa and Asia.
- Published 10's of papers in international journals and conferences.
- Wrote books on Experimental Systems Control and History of Mechanical Engineering.

