SEALS IN HIGH PRESSURE PLUNGER PUMPS: A MATHEMATICAL APPROACH TO LEAK PROOF SEAL DESIGN

Umesh Wazir
Mechanical Engineering ADE
University of Petroleum & Energy Studies, Bidholi
Dehradun, 248 007, Uttarakhand – India

ABSTRACT
High pressure plunger pumps generally use fixed contour metal seals for sealing the inlet and the outlet areas during its working cycle. Pressures to be sealed are high and pulsating in nature. Assembly and in situ service requirements necessitate a radial expanding seal design; which is preloaded to generate enough radial pressure so that there is no leakage and at the same time control its expansion so as not deform into plastic range and thus hinder seal extraction in service. The objective of this paper is to put forward, a mathematical relationship between clamp load and radial / axial bearing pressure which allows design engineer to assess effect of radial play, friction, seal wedge angle and the preload requirement. A design engineer will be able to decide on dimensions and the taper angle along with the bolt system dimensions at a very early stage of design and be sure of his design concept. Seal size determines the pump throughput and the system maximum pressure and thus the pump rating. Hence importance of this simple and effective evaluation methodology.

KEYWORDS: Radial seal expansion, Plunger metal seal, Plunger Pump, Sealing

I. INTRODUCTION
The requirements for a seal in a given application are multifaceted. Seals could be Contact, Non-contact; Fixed Contour solids or Flexible Elastomers. They could be operating in Static, Dynamic or Elasto Hydrodynamic conditions. However, what is demanded from all sealing systems, is, that under the given operating conditions it fulfils its objective of providing system safety at commercially competitive costing for an optimum sealing solution.

High pressure hydraulic plunger pumps (+500 bar) use solid metal seals for ensuring sealing between the pressure cylinder and inlet / outlet valve systems. They seal both in axial and radial direction under a known preload Fig. 1 If external clamp or preload is high and the seal undergoes localized plastic deformation, it becomes a maintenance nightmare to remove the seal without damaging the polished cylindrical ID. On the other hand if preload relaxes or is not adequate, the possibility of seal failing, by allowing leakage past the seal, is obvious. To meet this dual objective of controlled expansion and leak proof sealing, a hypothesis, using simplified form of ring element under internal and external load, is proposed so as to:

a) Bring out mathematical relationship between the external preload and radial displacement and bearing pressure, of radial expanding seals and
b) Use the finite element analysis technique to correlate the variances in the hypotheses

1.1. Current Published Information
As most of relayed applications lie in the realm of patents and copy rights [1], little published information was available on the behaviour of radial expanding seals, which took into account, assembly play, friction and radial bearing pressure generation. Most published work related to the area of seals in high pressure tubing and fittings. They primarily dealt with seal performance under varying conditions and effect of use of different materials. It has been found that perfect sealing is
achieved after complete plastic deformation of the seal at the joint interface takes place. Sealing then is independent of the surface finish quality and the fluid medium to be sealed. Lehmann [2] also differentiates between critical internal pressure (when the medium just starts to seep) and the blow-by pressure at which then the seal fails. Siebel and Raible [3] analysed the effect of da/di (outer to inner diameter ratio) of various seals on their performance behaviour. They concluded that the sealing behaviour did not change as long as the ratio da/di remained same.

From experimental evaluation [4] it has been concluded that, in general, for a seal to be effective, the clamp load i.e. the external load, must be three times greater than force resulting because of the internal pressure to be sealed. The results from [4] however must be considered, taking into account the seal design and the application.

Author assumes that a similar result for radial bearing pressure is expected. A typical plunger seal is generally sandwiched between the valve seat and the taper body. The preload $F_{ges}$ is exerted by the bolt arrangement. Fig 1 shows assembly of a typical seal used in such arrangements.

II. MATHEMATICAL MODEL DEVELOPMENT

2.1. Force Polygon

The axial Load $F_{ges}$ is directed to the seal through the valve body. For ease of understanding it is considered to be acting at the seal mid as a uniformly distributed line load along the circumference. The resulting components in the seal plane are shown in Fig 2 & 3, wherein $F_n$ is the normal force perpendicular to the parting plane, $\mu F_n$ and $\mu F_{ges}$ frictional forces opposing the radial displacement of the sealing ring.

The Force polygon in vertical direction gives

$$\mu F_n \frac{d_2}{2\pi r_m} \sin \alpha + F_n \frac{d_2}{2\pi r_m} \cos \alpha - F_{ges} \frac{d_2}{2\pi r_m} = 0$$

(1)
And solving for Radial and vertical direction forces

\[ F_r = F_{ges} \cdot \frac{\sin \alpha - \mu \cos \alpha}{\mu \sin \alpha + \cos \alpha} \]  
\[ \text{and,} \quad F_z = F_{ges} \]  

(2) \hspace{1cm} (3)

2.2 Basic Equations

Fig 4  Simplified seal section

If on a uniform thickness ring, bearing pressures \( p_a \) and \( p_i \) act on the inner and the outer surface ref Fig 4, then the radial displacement, based on Kantorowich [5], is given by the equation:

\[ u_p = \frac{1}{E} \left( \frac{1}{r^2} + \frac{r^2}{P_a + P_i} \right) \cdot \frac{1}{E} \left( \frac{1}{r^2} + \frac{r^2}{P_a + P_i} \right) \]  

(4)

Fig 6  & 7 Ring under radial loads

2.2.1 Case I : Hypothesis neglecting friction; \( \mu = 0 \)

Considering \( \mu = 0 \) we get from Eqn (2)

\[ F_{r, \mu=0} = F_{ges} \cdot \tan \alpha \]  

(5)

and therefore

\[ P = \frac{F_r}{2\pi r_m} = \frac{F_{ges} \cdot \tan \alpha}{2\pi r_m} \]  

(6)
This outwardly directed radial Line load $P$, we propose, is acting at mean radius $r_m$ ($r_i < r_m < r_a$). Using Eqn (4) we consider a case where the basic seal ring cut in two parts, whose radii are $r_i/r_m$ for the inner ring and $r_m/r_a$ for the outer ring, with $\delta$ being the ring thickness at the mean radius $r_m$. We consider that a part of total line load $P$, say $P_1$ acts on the inner ring and the balance $P_2 = P - P_1$ on the outer ring as shown in Fig 6. This gives us:

$P = P_1 + P_2$  \hspace{1cm} (7)

The radial displacement $u_1$ of the outer circumference of the inner ring must be same as the displacement $u_2$ of the inner circumference of the outer ring. Thus:

$u_1 = u_2 = u_{m}$  \hspace{1cm} (8)

Based on above conditions (Eqns 7 & 8) $P_1$ and $P_2$ can now be calculated. Under these conditions outer ring with internal bearing pressure $P_2/\delta$ (= $p_i$) can be examined. Now, should an external radial pressure $p_{ra}$ (in our case as a result of prevention of expansion = sealing) act on this outer ring, can be equated to radial play $u_{ra}^*$ between sealing ring and cylinder, as depicted in Fig 7.

$u_{ra}^* = f(p,p_{ra})$  \hspace{1cm} (9)

- **Basic Equation - Inner Ring Displacement**

The expansion $u_1$ of the outer diameter of the inner ring (Fig 6) because of the external pressure $p_a = P_1/\delta$ can be calculated with:

$p_a = \frac{P_1}{\delta}, \quad p_i = 0, \quad R = r_m, \quad r = r_i, \quad \rho = r_m$  \hspace{1cm} (10)

Substituting and simplifying the values in Eqn (4), we get:

$u_1 = \frac{(P_1/\delta) r_{m}^2}{(r_{m}^2 - r_{i}^2)} \cdot r_m \cdot \left[ (1 - v) + (1 + v) \frac{r_i^2}{r_m^2} \right]$  \hspace{1cm} (11)

- **Basic Equation - Outer Ring Displacement**

Similarly expansion $u_2$ (= $u_{m}$) of outer circumference of inner ring based on external stress $p_i = P_2/\delta$ and $p_a = -p_{ra}$ can be calculated with:

$p_i = \frac{P_2}{\delta}, \quad p_{ra} = -p_{ra}, \quad R = r_a, \quad r = r_m, \quad \rho = r_m$  \hspace{1cm} (12)

Substituting the values in Eqn (4), we get $u_2$ and similarly expression for $u_{ra}^* / u_{ra}$:

$u_2 = \frac{1-v}{E} \left( \frac{-p_{ra} r_{a}^2 + \frac{P_2}{\delta} r_{m}^2}{r_{a}^2 - r_{m}^2} \right) \cdot r_m + \frac{1+v}{E} \left( \frac{-p_{ra} r_{a}^2 + \frac{P_2}{\delta} r_{m}^2}{r_{a}^2 - r_{m}^2} \right) \cdot \frac{1}{r_m}$  \hspace{1cm} (13)

$u_{ra}^* = \frac{1-v}{E} \left( \frac{-p_{ra} r_{a}^2 + \frac{P_2}{\delta} r_{m}^2}{r_{a}^2 - r_{m}^2} \right) \cdot r_a + \frac{1+v}{E} \left( \frac{-p_{ra} r_{a}^2 + \frac{P_2}{\delta} r_{m}^2}{r_{a}^2 - r_{m}^2} \right) \cdot \frac{1}{r_a}$  \hspace{1cm} (14)

- **Basic Equations - Radial Bearing Pressure**

From Eqn (8) we get the expression:

$\frac{(p_{ra}/\delta) r_{m}^2}{E (r_{a}^2 - r_{m}^2)} \cdot \left[ (1 - v) r_{m}^2 + (1 + v) r_{a}^2 \right] = \frac{1-v}{E} \left( -p_{ra} r_{a}^2 + \frac{P_2}{\delta} r_{m}^2 \right) r_m + \frac{1+v}{E} \left( -p_{ra} r_{a}^2 + \frac{P_2}{\delta} r_{m}^2 \right) r_a$  \hspace{1cm} (15)

The three equations Eqn (7), (14) and (15) contain three unknowns, namely $P_1$, $P_2$ and $p_{ra}^*$. Substituting suitably and simplifying we get:

$\frac{P_2}{E} \left( \frac{(1-v) r_{a}^2 + (1+v) r_{i}^2}{(r_{a}^2 - r_{i}^2)} \right) = \frac{p_{ra}^*}{E} \left[ \frac{(1-v) r_{a}^2 + (1+v) r_{i}^2}{(r_{a}^2 - r_{i}^2)} \right] \cdot r_a$  \hspace{1cm} (16)

Fig 8 Considering frictional force

2.2.2 Case II: Hypothesis Considering frictional resistance during radial displacement
In Eqn (16) calculated bearing pressure and displacement of ring outer profile are under the conditions when friction \( \mu = 0 \). Should the friction be considered then radial acting force \( F_{r} \) gets reduced by an amount \( Fr' \), where \( Fr' \) is given by:

\[
Fr' = (-) \mu \cdot F_{ges}
\]

Thus, taking \( \mu \) into account, the effective radial line load, acting on median circumference, \( P_{eff} \) is:

\[
P_{eff} = \frac{F_{r}}{2\pi r_{m}} - \frac{Fr'}{2\pi r_{m}}
\]

(17)

\[
P_{eff} = P \cdot \frac{1-\mu^{2} - 2\mu \cot \alpha}{1 + \mu \tan \alpha}
\]

(18)

By meeting the condition:

\[
1 - \mu^{2} - 2\mu \cot \alpha \geq 0
\]

(19)

ie \( \alpha \geq \arctan \left( \frac{2\mu}{1 - \mu^{2}} \right) \)

it is now possible to evaluate the radial bearing pressure \( p_{rea} \) under the influence of frictional resistance where now in Eqn (16) \( P \) is simply to be replaced by \( P_{eff} \).

From Eqn (19) it is also clear that for small values of \( \alpha \) the radial displacement does not occur ie. in such a case only the lateral expansion is to be considered.

2.2.3 Case III: Hypotheses Considering effect of lateral deformation

The effect of lateral elongation as a result of the vertical load \( F_{z} = F_{ges} \) (Eqn 2) can be approximated by a conditional consideration: ie considering a unidirectional stress state ie elongation through deformation of the seal as a whole neglecting any effect of friction.

The actual expansion as a result of lateral deformation will be smaller because of the presence of friction. Thus the actual value of radial expansion will lie between the results calculated considering and not considering lateral deformation.
Further, is the ring free to expand without resistance, we can consider it as a protracted case of unidirectional state of stress.

Thus, referring Fig 9; With $\sigma_\phi = \sigma_r = 0$ we get

In radial direction,
$$\varepsilon_r = \left(-\frac{\sigma_r}{E}\right) \cdot \nu$$

(20)

And in circumference direction
$$\varepsilon_\phi = \left(-\frac{\sigma_\phi}{E}\right) \cdot \nu$$

(21)

$$\Delta r_m = \varepsilon_\phi \cdot r_m \quad \Delta b = \varepsilon_r \cdot b$$

(22)

The lateral deformation $\bar{u}_{ra}$ thus equals

$$\bar{u}_{ra} = \nu \cdot \frac{r_m}{\pi E} \cdot \frac{F_{ges}}{r_a^2 - r_i^2}$$

(23)

### 2.3 Mathematical Model Hypothesis - Summary

Thus incorporating the lateral deformation $\bar{u}_{ra}$ Eqn (23) and frictional resistance $\mu$ through $P_{eff}$ Eqn (18) in the basic Eqn (16) and simplifying we get expression Eqn (24). This allows us to evaluate $u_{ra}$ ($u_{ra}^*$) or $p_{ra}^*$

ie From Hypotheses I + II + III, we get the mathematical model as:

$$\frac{r_a}{E} \cdot \frac{P_{eff}}{\delta} \cdot \frac{(1-\nu) r_m^2 + (1+\nu) r_i^2}{(r_a^2 - r_i^2)} + \bar{u}_{ra} - u_{ra}^* = \frac{p_{ra}^*}{E} \cdot \frac{[(1-\nu) r_m^2 + (1+\nu) r_i^2]}{(r_a^2 - r_i^2)} \cdot r_a$$

(24)

### III. FEM SIMULATION RESULTS

A simulation [1] with finite element method using a TRIAX element was conducted with different ring shapes. FEM results (rated comparison) is summarised and tabulated below in Fig 10. Also given there in is the result using the computational mathematical model as summarised in the hypotheses Eqn (24).

<table>
<thead>
<tr>
<th>Ring No</th>
<th>Material</th>
<th>$h$</th>
<th>$S \times 45^\circ$</th>
<th>$\alpha$</th>
<th>$\delta$</th>
<th>$u_0$ = Expansion / Ton load (Fges)</th>
<th>$p_0$ = Radial Pressure / Ton load (Fges)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>FEM micron/Ton</td>
<td>EQN 24 micron/Ton</td>
</tr>
<tr>
<td>1</td>
<td>Mild Steel</td>
<td>9</td>
<td>2</td>
<td>45</td>
<td>6.5</td>
<td>5.2</td>
<td>4.3</td>
</tr>
<tr>
<td>2</td>
<td>Mild Steel</td>
<td>9.5</td>
<td>1</td>
<td>45</td>
<td>6.7</td>
<td>5.3</td>
<td>4.5</td>
</tr>
<tr>
<td>3</td>
<td>Mild Steel</td>
<td>9</td>
<td>0.2</td>
<td>45</td>
<td>6.5</td>
<td>4.7</td>
<td>4.3</td>
</tr>
</tbody>
</table>

Fig 10 FEM and Computational results for rated Radial expansion $u_0$ and Radial pressure $p_0$ Tabulated
The actual displacement profile and the profile of radial pressure generated under known load conditions are shown in Figs 11 and 12.

3.1 FEM and Mathematical Model Result Discussion
A quick FEM comparison shows a fairly good overall relationship as far as radial expansion $u_0$ and its distribution is concerned. The radial pressure mean value is in consonance with the calculated value but its distribution does shows its dependence on the leading edge chamfer. It is to be assumed that the edge chamfer, ‘S’ plays an important role in $p_0$ generation in such design applications.

- $u_0$ ie Radial Expansion / Ton load (Fges) shows a very good overall matching of the values both @ $d_m$ as well as distribution over $\delta$
- $p_0$ ie Radial Pressure / Ton load (Fges) shows a very good overall matching of the values @ $d_m$ but the distribution over $\delta$ shows that the leading edge chamfer Sx450 plays a role.

Fig 11 & 12 show plotted summary of the FINEL analysis. This calculation was done with Fges = 4905 daN, $E = 20.6 \times 10^4$ N/mm2, $\mu = 0.1$ and $\nu = 0.3$

IV. CONCLUSIONS
4.1 Mathematical Model
The comparative results show a good overall relationship and the mathematical model derived under Ch 2.0 can be used for a direct evaluation of radial expansion and radial pressure generated under different external conditions as summarised:

\[ P_{\text{eff}} = \frac{F_{\text{ges}} \tan \alpha}{2 \pi r_a} \cdot [(1 - \mu^2 - 2 \mu \cot \alpha)/(1 + \mu \tan \alpha)] \]  
[where by \( \alpha \geq \arctan (2\mu/1 - \mu^2) \)]

\[ \bar{u}_{r \alpha} = \nu \cdot \frac{\tan \gamma}{\pi E} \cdot \frac{F_{\text{ges}}}{r_a^2 - r_i^2} \]  
(26)

and \( r_m = (r_a + r_i)/2 \)

- By unhindered i.e. free expansion the radial expansion \( u_{r \alpha}^\circ (= u_{r \alpha}^f) \)

\[ u_{r \alpha}^f = \frac{P_{\text{eff}}}{\delta} \cdot \frac{r_a}{E} \cdot \frac{1}{r_a^2 + r_i^2} \]  
(28)

- For a given radial play \( u_{r \alpha}^a \) (where by \( u_{r \alpha}^a > u_{r \alpha}^f \)) the maximum possible radial contact pressure \( p_{r \alpha}^a \)

\[ p_{r \alpha}^a = \frac{P_{\text{eff}}}{\delta} \cdot \frac{[(1 - \nu) r_a^2 + (1 + \nu) r_i^2]}{[(1 - \nu) r_a^2 + (1 + \nu) r_i^2]} \]  
(29)

### 4.2 Future Direction

Based on FEM evaluation it is seen that leading edge base chamfer ‘S’ , plays an important role in pressure distribution profile. Also the definition of \( \delta \) and \( d_m \) can be further refined for its sensitivity. Thus further work that could be carried out encompasses:

- Effect Of Base Chamfer and its cross correlation with the FEM results
- Sensitivity analysis of \( \delta \) and \( d_m \) on the results \( u_{r \alpha}^a \) & \( p_{r \alpha}^a \)
- Experimental measurement of radial deformation and pressure generated.

### REFERENCES

[1]. Umesh Wazir; Experimental Untersuchung von Pumpendichtungen; Diplomarbeit; TH Stuttgart 1977
[2]. Lehmann ; Dissertation “ Flachdichtungen” TU Berlin 1953
[4]. Siebel u. Raible Einfluss der Probenabmessungen Mitteilung VBG-
[5]. Kantorowitsch, KB, Die Festigkeit der Maschinen in der chemischen Industrie VEB –Verlag Berlin 1953
[6]. Sabo,G Technical Mechanics Springer Verlag Berlin -1972
[7]. Biezeno u. Grammel ; Technical Dynamics Springer Verlag Berlin -1971
[8]. Zienkiewicz, OB, Finite Element Method – Hauser verlag, Munich 1975

### Symbols used

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Units</th>
<th>Nomenclature</th>
</tr>
</thead>
<tbody>
<tr>
<td>b</td>
<td>mm</td>
<td>Sealing ring width</td>
</tr>
<tr>
<td>d_a</td>
<td>mm</td>
<td>outer diameter</td>
</tr>
<tr>
<td>d_m</td>
<td>mm</td>
<td>median diameter</td>
</tr>
<tr>
<td>d_i</td>
<td>mm</td>
<td>inner diameter</td>
</tr>
<tr>
<td>E</td>
<td>N/mm^2</td>
<td>modulus of elasticity</td>
</tr>
<tr>
<td>F_{ges 1}</td>
<td>N</td>
<td>clamp force , expansion</td>
</tr>
<tr>
<td>F_{ges 2}</td>
<td>N</td>
<td>clamp force , contact pressure</td>
</tr>
<tr>
<td>F_{ges}</td>
<td>N</td>
<td>Total Clamp force</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Units</th>
<th>Nomenclature</th>
</tr>
</thead>
<tbody>
<tr>
<td>s</td>
<td>mm</td>
<td>ring base chamfer</td>
</tr>
<tr>
<td>S_f</td>
<td>-</td>
<td>safety factor</td>
</tr>
<tr>
<td>u_\rho</td>
<td>mm</td>
<td>radial displacement @ ( r = \rho )</td>
</tr>
<tr>
<td>u_m</td>
<td>mm</td>
<td>radial displacement @ ( r = m )</td>
</tr>
<tr>
<td>u_{1,2}</td>
<td>mm</td>
<td>radial displacement @ ring 1,2</td>
</tr>
<tr>
<td>\bar{u}_{r \alpha}</td>
<td>mm</td>
<td>lateral expansion</td>
</tr>
<tr>
<td>\bar{u}_{r \alpha}^\circ</td>
<td>mm</td>
<td>Radial displacement @ ( \bar{u}_{r \alpha} = 0 )</td>
</tr>
<tr>
<td>\bar{u}_{r \alpha}^\circ</td>
<td>mm</td>
<td>Radial displacement, total</td>
</tr>
</tbody>
</table>
**Authors Biography**

Umesh Wazir was born in India in 1949. He received his bachelor’s degree from National Institute of Technology Warangal (India) in 1969-70 and his Masters from the Technical University Stuttgart (Germany) in the year 1977-78. Mr Wazir has an experience of 42 yrs; 22 yrs in R&D with Mercedes Benz (Ger), Escorts Ltd (India); 12 yrs in Operations with Piaggio (India) as Director Manufacturing, and Head Product Support Escorts (India); and 8 yrs in Academics. His research interests include Sealing Technology, Gear Design Optimization, New Material application and Vehicle Design Engineering.