POSITION ANALYSIS BASED APPROACH FOR TRAJECTORY TRACKING CONTROL OF SCORBOT-ER-V PLUS ROBOT MANIPULATOR

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ABSTRACT

One of the most common requirements in robotics is to move the end-effector smoothly from initial location to goal location. In this paper, two approaches to generating such trajectories: straight lines in joint space and straight lines in Cartesian space have been discussed. These are known respectively as joint space and Cartesian space trajectory tracking. Two user defined algorithms are developed for joint space as well as Cartesian space trajectory tracking. The algorithm has been tested in simulation yielding fair results, which have also been compared with each other to check the accuracy and results of both.

KEYWORDS: Dof, Matlab, Trajectory Planning, Smoothness, Singularities Avoidance, Cartesian Path Generation,

I. INTRODUCTION

Position analysis based solutions are very vital when one wants to perform modelling of robotic arm. It turns out to be a difficult task to find the solution through inverse kinematics with increase in Degree of Freedom (DOF) of robot. The conventional methods used for calculating inverse kinematics of any robot manipulator are: geometric [1-2], algebraic [3-5] and iterative [6] approaches. While algebraic methods cannot promise closed form solutions. Geometric methods must be able to produce closed form solutions for the first three joints of the manipulator geometrically. The iterative methods on the other hand approach only to a single solution and that solution also depends on the starting point. To solve the inverse kinematics problem for three different cases of three DOF manipulator in three dimensional spaces, a solution was proposed in [7] using feed-forward neural networks. This introduces the fault-tolerant and highspeed advantages of neural networks to the inverse kinematics problem. A three-layer partially recurrent neural network was proposed by [8] for trajectory planning and to solve the inverse kinematics as well as the inverse dynamics problems in a single processing stage for the PUMA 560 manipulator. Hierarchical control technique based on the establishment of a non-linear mapping between Cartesian and joint coordinates using fuzzy logic in order to direct each individual joint was proposed in [9] for controlling a robotic manipulator. Commercial Microbot with 3-DOF was utilized to evaluate the proposed method. A novel modular neural network system to overcome the discontinuity of the inverse kinematics function was proposed in [10] that consist of a number of expert neural networks. Neural network based three-joint robotic manipulator simulation software was developed in [11] for inverse kinematics solution of a robotic manipulator. Then a designed neural network was used to solve the inverse kinematics problem. An Artificial Neural Network (ANN) based on Bees Algorithm using backpropagation algorithm was applied in [12] to solve inverse kinematics problems of industrial robot manipulator. That in turns used to train multi-layer perceptron neural networks in [13] to model the inverse kinematics of an articulated robot manipulator arm. An
Artificial Neural Network (ANN) based approach for fast inverse kinematics computation and effective geometrically bounded singularities prevention of redundant manipulators was presented in [14]. First some bounded geometrical concepts were used to establish some characterizing matrices, to get a simple performance index, and a null space vector for singularities avoidance/prevention and secure way generation. Then inverse kinematics based on above assumptions was computed using a properly trained ANN. A reliability neural network based inverse kinematics solution approach was presented in [15], which in turns applied to a 6-DOF robot manipulator. The structure of the proposed method was based on Elman network using three networks designed parallel to minimize the error of the whole system. An adaptive learning strategy using an artificial neural network (ANN) to control the motion of a 6-DOF manipulator robot by overcoming the inverse kinematics problem was proposed in [16] which mainly included singularities and uncertainties in arm configurations. The proposed control technique did not require any prior knowledge of the kinematics model of the system being controlled. An Adaptive NeuroFuzzy Inference System (ANFIS) method based on the Artificial Neural Network (ANN) was applied in [17] to design an Inverse Kinematic based controller for the inverse kinematical control of SCORBOT-ER-V Plus. The proposed ANFIS controller combined the advantages of a fuzzy controller as well as the quick response and adaptability nature of an ANN. Computer Simulation was carried out to demonstrate the accuracy of the proposed controller to generate an appropriate joint angle for reaching desired Cartesian state, without any error.

A path tracking algorithm to compensate for path deviation due to torque limits was proposed by [18] which in turns used a disturbance observer to obtain a Simple Equivalent Robot Dynamic (SERD) model to modify the desired acceleration of the nominal trajectory in Cartesian space. A technique based on Continuous Genetic Algorithms (CGA) to solve the path generation problem for robot manipulators was presented in [19]. The inverse kinematics problem was formulated as an optimization problem based on the concept of the minimization of the accumulative path deviation and is then solved by the presented technique. A unified approach to optimal pose trajectory planning for robot manipulators in Cartesian space through a genetic algorithm (GA) enhanced optimization of the pose ruled surface was presented in [20]. The optimization model is established based on functional analysis and dynamics planning, and instantiated by using higher order parametric space curves as position and orientation trajectories. [21] discussed the problem of minimum cost trajectory planning for robotic manipulators, in which the generic optimal control problem was transformed into a non-linear constrained optimization problem which is treated then by the Sequential Quadratic Programming (SQP) method. Simulated Annealing (SA) technique was applied to the problem of robot path planning in [22]. Three situations were considered: the path is represented as a polyline; as a Bézier curve; and, as a spline interpolated curve. In order to ensure that the resulting trajectory is smooth enough, an objective function containing a term proportional to the integral of the squared jerk (defined as the derivative of the acceleration) along the trajectory was considered in [23]. Then another term proportional to the total execution time was added to the expression of the objective function. Fifth order B-splines were then used to compose the overall trajectory. The path planning problem with general end-effector constraints for robot manipulators was addressed in [24]. Two approaches were proposed. The first approach was the Adapted Randomized Gradient Descent (ARGD) method, and the second approach was radically different as it worked in both task space and Cartesian space in comparison with first which searches purely in Cartesian space. A planning model of trajectory motion for serial link manipulators with higher degree polynomials application was discussed in [25]. The linear acceleration profiles of end-effector, for each coordinate, were planned as the polynomials of degrees 9, 7 and 5. This approach to polynomial form structure necessitates the determination of only one polynomial coefficient, irrespective of its order. Method based on trajectory planning to avoid the detachment of joint elements of a manipulator with clearances was offered in [26]. An improved detachment criterion for the revolute and spherical joints was proposed. An analysis based on cubic splines or fifth order B-splines trajectory of an algorithm for optimal trajectory planning of robot manipulators was presented in [27]. The objective function to be minimized is a weighted sum of the integral squared jerk and the execution time. The proposed technique allowed setting constraints on the robot motion, expressed as upper bounds on the absolute values of velocity, acceleration and jerk. A robust and fast procedure that could be used to identify the joint stiffness values of any six revolute serial robots was presented in [28]. The proposed method aimed to evaluate joint stiffness values by considering both translational and rotational displacements of the robot end-
effector for a given applied wrench (force and torque). An experimental investigation on a robot manipulator, using neural network for analyzing the vibration condition on joints was done in [29]. Firstly, robot manipulator’s joints were tested with prescribed of trajectory end-effectors for the different joints speeds. Furthermore, noise and vibration of each joint were measured. And then, the related parameters were tested with neural network predictor to predict servicing period. While exposing some of the issues associated with shortcuts-like post-processing algorithms, the [30] addressed the optimization of paths generated using an iterative user defined randomized algorithms.[31] proposed a new method for inverse kinematics for hyper-redundant manipulators, to plan the path of the end-effector. The inverse kinematics was computed for a given smooth path consisting of points close enough to each other.[32] proposed a new approach for solving the problem of obstacle avoidance during manipulation tasks performed by redundant manipulators, which solution is based on a double neural network that uses Q-learning reinforcement technique. Q-learning has been applied in robotics for attaining obstacle free navigation or computing path planning problems.

In this paper, we present a simulation based solution for trajectory tracking of SCORBOT-ER-V PLUS robot manipulator. It can efficiently solve practical and real size problems. Robot positional analysis singularities and workspace constraints were considered in developing the model. The method was tested on a SCORBOT-ER-V PLUS robot in the Control System and Robotics Laboratory at IIT Roorkee.

This paper is organized into four sections. In the next section, the Positional analysis (Forward as well as inverse kinematics) of SCORBOT-ER-V Plus has been briefed with the help of DH algorithm as well as conventional techniques methods. The trajectory planning based on the positional analysis is introduced in section 2. It also explains about the two approaches used for trajectory tracking with the help of flow charts. Simulation results are discussed in section 3. Section 4 gives concluding remarks.

II. POSITION ANALYSIS OF SCORBOT-ER-V PLUS [17]

SCORBOT-ER-V Plus is a vertical articulated robot, with five revolute joints. It has a Stationary base, shoulder, elbow, tool pitch and tool roll. Figure 1.1 shows the schematic diagram of SCORBOT-ER-V Plus Robot manipulator.

For the kinematic model of SCORBOT first we have to assign frame to each link starting from base (frame {0}) to end-effector (frame {5}). The frame assignment is shown in Figure 1.1. The corresponding link parameters are listed in Table 1.1.

<table>
<thead>
<tr>
<th>Joint i</th>
<th>ai (mm)</th>
<th>al (mm)</th>
<th>θi</th>
<th>Operating range</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>- π/2</td>
<td>16</td>
<td>349</td>
<td>-155° to +155°</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>221</td>
<td>0</td>
<td>-35° to +130°</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>221</td>
<td>0</td>
<td>-130° to +130°</td>
</tr>
<tr>
<td>4</td>
<td>-π/2</td>
<td>0</td>
<td>0</td>
<td>π/2 + θ4</td>
</tr>
<tr>
<td>5</td>
<td>0</td>
<td>0</td>
<td>145</td>
<td>-570° to 570°</td>
</tr>
</tbody>
</table>
2.1. Forward Kinematic of SCORBOT–ER-V Plus

Once the DH coordinate system has been established for each link, a homogeneous transformation matrix can easily be developed considering frame \{i-1\} and frame \{i\}. This transformation consists of four basic transformations.

\[
0T_5 = 0T_1 * 1T_2 * 2T_3 * 3T_4 * 4T_5
\]  

Finally, the transformation matrix is as follow:

\[
T = \begin{bmatrix}
-S_1S_5 - C_1S_2S_5 & -C_1S_5 + C_1C_2S_5 & C_2C_1 \quad & C_1(a_1 + a_2C_3 + a_3C_5 + dC_5) \\
C_1S_5 & C_2S_1 + C_1S_2S_1 & -S_1 & S_2(a_1 + a_2C_3 + a_3C_5 + dC_5) \\
-C_2S_5 & S_1S_2S_5 & -S_2 & (d_1 - a_2S_1 - a_3S_3 - dS_3) \\
0 & 0 & 0 & 1
\end{bmatrix}
\]  

Where, \( S_i = (\theta_i) \), \( C_i = (\theta_i) \) and \( S_{ijk} = (\theta_i + \theta_j + \theta_k) \). \( C_{ijk} = (\theta_i + \theta_j + \theta_k) \).

The \( T \) is all over transformation matrix of kinematic model of SCORBOT-ER-V Plus, from this we have to extract position and orientation of end-effector with respect to base is done in the following section. As SCORBOT-ER-V is having 5 DOF, only five of the six of end-effector parameters can be specified.

2.2. Obtaining Position in Cartesian Space

The value of \( x, y, z \) is found from last column of transformation matrix as:

\[
X = C_1(a_1 + a_2C_2 + a_3C_3 + d_1C_334) \\
Y = S_1(a_1 + a_2C_2 + a_3C_3 - d_5C_334) \\
Z = (d_1 - a_2S_1 - a_3S_3 - dS_334)
\]  

**Pitch:** Pitch is the angle of rotation about y axis of end-effector

\[
pitch\beta = \theta_2 + \theta_3 + \theta_4 = \theta_{234} \\
\theta_{234} = a \tan 2(r13, \pm \sqrt{r23^2 + r33^2})
\]  

Here we use atan2 because its range is \([-\pi, \pi]\), where the range of atan is \([-\pi/2, \pi/2]\).

**Roll:** The roll is derived as follow:

\[
\theta_2 = a \tan 2(r12 / C_{234}, r11 / C_{234})
\]  

**Yaw:** Here for SCORBOT yaw is not free and bounded by \( \theta_1 \).

2.3. Inverse Kinematics of SCORBOT-ER-V Plus

For SCORBOT we have five parameter in Cartesian space is \( x, y, z \), roll (\( \beta \)), pitch (\( \gamma \)). For joint parameter evaluation we have to construct transformation matrix from five parameters in Cartesian coordinate space. For that rotation matrix is generated which is depends on only roll, pitch and yaw of robotic arm. For SCORBOT there is no yaw but it is the rotation of first joint \( \theta_1 \).

So the calculation of yaw is as follow:

\[
\alpha = \theta_1 = a \tan 2(x, y)
\]  

So, the total transformation matrix is as follows:
\[
T = \begin{bmatrix}
-S_\beta S_\alpha - C_\alpha C_\beta S_\gamma & -S_\beta C_\alpha + S_\alpha C_\beta S_\gamma & C_\beta C_\gamma & X \\
C_\beta S_\alpha - C_\alpha S_\beta S_\gamma & C_\beta C_\alpha + S_\beta S_\gamma S_\alpha & C_\gamma S_\beta & Y \\
-C_\alpha C_\gamma & C_\gamma S_\alpha & -S_\gamma & Z \\
0 & 0 & 0 & 1
\end{bmatrix}
\] (10)

After comparing the transformation matrix in equation (2) with matrix in equation (14), one can deduce:

\[
\begin{align*}
\theta_1 &= \alpha, \\
\theta_{234} &= \beta, \\
\theta_5 &= \gamma, \\
\theta_3 &= a \tan (\pm \sqrt{1 - \cos^2 \theta_3}, \cos \theta_3)
\end{align*}
\] (11)

\[
\theta_2 = -a \tan (Y_3, X_3) - a \tan (a_1 \sin \theta_3, a_2 + \cos \theta_3)
\] (12)

Finally we will get:

\[
\theta_4 = \theta_{234} - \theta_2 - \theta_3
\] (13)

### III. TRAJECTORY CONTROL

A trajectory is the track followed by the manipulator, plus the time profile along the path. Trajectories can be calculated either in joint space (directly specifying the time evolution of the joint angles) or in Cartesian space (specifying the position and orientation of the end frame). Matters in trajectory planning include attaining a specific target from an initial starting point, avoiding obstacles, and staying within manipulator capabilities. Planning in joint space is the simplest and fastest, because inverse kinematics is avoided. The shortcoming is that the end-effector posture is not directly controlled, and hence collision avoidance is challenging. Planning in Cartesian space allows being met the geometric constraints of the external world, but only after solving inverse kinematics. Two algorithms were developed to test the trajectory tracking based on positional analysis.

In the following section a description of the methodology somewhat interactive, developed using MATLAB is given for both types of algorithms.

#### 3.1 Description of Interactive Methodology

Figures 1.2 as well as Figure 1.3 give flowcharts of the capabilities of the software developed using MATLAB. The software has the capability to calculate joint space as well as Cartesian space trajectories for the SCORBOT-ER-V Plus Robot manipulator while calculating forward as well as inverse kinematics integrated into one interactive package. At each menu driven step of the identification procedure, the user chooses the next step to be pursued. For clarity the program sequence steps are explained below.

1) The first step is to select one of the options from menu in front of user. Here user has the opportunity to choose either joint space trajectory generation or Cartesian space trajectory generation option from the Graphical User Input (GUI) based menu.

2) The next step is to calculate forward as well as inverse kinematics solution from initial point to the final point. Then total time is assigned using maximal velocities in joints. Afterwards the individual joint trajectories in time are discretized by the program if the joint space trajectory option has been selected. Later on a continuous function is blended between these points.

3) In the next step it checks for number of input arguments and based on the decision it normalizes the time. Then on the basis of number of output arguments it computes velocity as well as acceleration. During the execution of the program at various stages if some of the conditions are not met then program also generates error exceptions.

4) If the user has chosen the second option i.e. Cartesian space trajectory generation option then after defining the initial as well as goal points, it checks for homogeneity of the input data. If data is homogenous it generates an error exception otherwise checks for time scaling.
5) If time is scaled then it makes the data a smooth function of time. On the other hand if time is not scaled then it understands that data is already a smooth function of time.

5) After execution of the program, user gets various graphs like given in Figure 1.4 to Figure 1.7 as well as Figure 1.8 to Figure 1.11 for both of the algorithms. From these graphs, a user can rationalize whether the proposed model is capable of reproducing the desired results or not.

6) In the sequence, program also generates the motion through singularity comparison response for the SCORBOT-ER-V Plus Robot manipulator, which ultimately helps a user to get the idea about the vulnerability through singular area in both the cases.

If the model is quite capable of reproducing the reference data and various graphs generated, shows that robot manipulator has not lost the DOF through singular area and the path followed by manipulator is smooth, then the results are satisfactory, otherwise the entire procedure could be repeated. All the results are stored in files which can be examined and the procedure could be repeated if required. Flow charts based on the established algorithms are given in Figure 1.2 as well as in Figure 1.3 as follows:

![Flow chart](image-url)
IV. SIMULATION RESULTS AND DISCUSSION

4.1 Joint Space Trajectory

The end-effector movement between two Cartesian poses which falls in the XY plane was considered. The orientation of the end-effector was taken downwardly. Time period for the whole exercise was of 2 seconds in 50ms of time steps. The joint, velocity and acceleration were attained as a function of time by designing a joint space trajectory for smoothly incorporating between two formations. The joint angles contrasted with time are revealed in Figure 1.4. Though it illustrates a smooth joint coordinate trajectory but unable to provide perfect evidence regarding Cartesian space end-effector trajectory. But by applying forward kinematics to the joint coordinate trajectory, it can be calculated easily.

![Joint Vector](image-url)

Figure 1.4: The Joint Coordinate Vector
The end-effector position in Cartesian space is plotted against time in Figure 1.5.

![End-effector Position in Cartesian space](image)

End-effector route in the XY plane is shown in Fig. 1.6 and one can easily predict that the track is not a straight line. This is to be anticipated due to the Cartesian nature of the end-points. Due to rotation around the waist, the end-effector will logically track a circular arc. It can cause accidents in reality among the robot and neighboring entities though they do not fall on the track between poses A and B.

![End-effector path in XY Plane](image)

The orientation of the end-effector in Euler angle form is plotted in Figure 1.7.

![End-effector Orientation](image)

4.2 Cartesian Space Trajectory

Straight-line motion for many applications is required in Cartesian space which is known as Cartesian motion. The equivalent joint-space trajectory by applying the inverse kinematics is acquired and is drawn in Figure 1.8.

![The Joint Coordinate Vector](image)
The end-effector position in Cartesian space is plotted against time in Figure 1.9.

![End-effector position in Cartesian space](image)

If Figure 1.6 and Figure 1.7 are compared, one can find certain significant dissimilarities. The position in Figure 1.10 as well as orientation presented in Figure 1.11 varies linearly with time. For orientation the pitch angle is constant at zero at start as well as end and varies negatively along the path. The end-effector tracks a straight line in the XY plane as presented in Figure 1.10.

The orientation of the end-effector in Euler angle form is plotted in Figure 1.11.

![End-effector orientation](image)

4.3 Motion through Singularity

The Cartesian trajectory endpoints taken previously were intentionally altered for a robot to travel in the y-direction with the end-effector z-axis pointing in the x-direction through singularity. Due to this, the manipulator has lost one degree of freedom and acts as 5-DOF manipulator. The jointspace tracking between the two postures marked in Figure 1.5 is invulnerable to this difficulty since it does not need the inverse kinematics, but it will sustain the posture of the tool in the x-direction only at the two end points, not for the entire pathway. The manipulability measure for the track was virtually zero about the time of the swift wrist joint motion, is displayed in Figure 1.12.
V. CONCLUSION

Two methodologies principally centered on joint space as well as Cartesian space tracking for tracking the end-effector effortlessly between dissimilar postures were presented. The precision of the output of the projected algorithms for effective positional tracking control of industrial robot can be experienced from the simulation results. Joint space trajectory may be challenging for some applications due to not result in Cartesian space straight line trajectory. Due to singularities in the workspace, though straight line Cartesian space trajectory can be established but may lead to very high joint rates. Collision avoidance is difficult due to non-operability of end-effector in case of joint space trajectory tracking. On the other to realize the geometric constraints of the external world in Cartesian space one has to solve the inverse kinematics. The methodology presented here can be extended to be used for trajectory planning and quite a few tracking applications based on positional analysis with real world disturbances. The present work did not make use of dynamics of robot manipulator, so it could be extended for the same also.

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