OPTIMAL BLOCK REPLACEMENT MODEL FOR AIR CONDITIONERS USING HIGHER ORDER MARKOV CHAINS WITH & WITHOUT INFLATION

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ABSTRACT

This paper deals with the development of a mathematical model for the replacement of a block of items using discrete-time higher order Markov Chains. In this study, a special reference has been given to a block of Air Conditioners. In order to make the model to yield more realistic results, three intermediate states viz., Minor Repair State, Semi-Major Repair State and Major Repair States have been considered between working State & breakdown State of the system. The model is developed with an objective of optimizing the maintenance costs associated with a block of similar multi state repairable items. Transition Probabilities for future periods are estimated by Spectral Decomposition Method for First Order Markov Chain (FOMC) and Moving Weighted Transition Probabilities (MWTP) Method for Second (Higher) Order Markov Chain (SOMC). Using these probabilities, the number of Air Conditioners in each state and the corresponding average maintenance costs are computed. The forecasted inflation for Air Conditioners and the real value of money using Fisherman's relation are employed to study and develop the real time mathematical model for block replacement decision making.

KEYWORDS: *Replacement, Higher Order Markov Chains, Transition Probability, Spectral Decomposition Method, Inflation.*

I. INTRODUCTION

The replacement decisions in most of the multinational companies, Software Development Centers, Star Hotels and other major food processing industries are mostly with the air conditioners. The primary decision is generally whether to replace the existing block of air conditioning system containing a large number of air conditioners or use for some more period otkgvk,f time.

The activity of maintenance involve a variety of repairs & services ranging from minor to major which cannot be defined and computed exactly in specific, various costs, and influence of various economic variables such as Inflation, value of money etc.

Several researchers investigated the optimal age replacement models with repairs to reduce the cost. Nuthall et *al* (1983)[18] studied the impact of inflation on replacement costs along with the impact of some other parameters viz. financing method and increased or decreased hours of use. Chein et *al* (2007) [5] presented an age-replacement model with minimal repair based on cumulative repair cost limit. In this they considered the complete repair cost data in order to decide whether to repair the unit or to replace. Bagai *et al* (1994) [9] discussed optimal replacement time under the age replacement policy for a system with minimal repair that involves the replacement of only a very small part of the system. Rupe et al (2000) [10] explored the maintenance models for finite time missions by considering net present value of costs.

There are some studies on the replacement decisions for warranted products. Zuo *et al* (2000) [28] discussed replacement policy for multi state Markov deterioration of machines that are under warranty. Pan *et al* (2010) [26]extended the work of Zuo *et al* (2000) [28] by considering more general state space with time parameters at each state.

Archibald *et al* (1996) [24] studied and compared the optimal age-Replacement, Standard Block Replacement and Modified Block Replacement (MBR) Policies with an inference of MBR policy is appreciably better than the remaining two.

However there is no much literature on block or group replacement model with Markov chain transition probabilities that is being used in many applications.

Markov chains concept was used by Stelios et al (1980) [22], Jianming et al (2003) [11], Shamsad et al (2005) [21], Ying-Zi Li et al (2009) [25], Avik Gosh et al (2010) [1], Carpinone et al (2010) [4], for forecasting of different parameters viz. Power generation, monsoon rainfall, manpower supplying, wind speed. With the increasing popularity of use of Markov chains, some studies by Bruce Craig et al (1998) [3] and Liana Cazacioc et al (2004) [12] are made on estimation and evaluation of transition probabilities using Markov chains.

Markov Chain forecasts as observed by Ying-Zi Li et al (2009) [25] have some practical value that yielded relatively satisfied results. Shamsad et al (2005) [21] and Carpinone et al (2010) [4] observed that Second order Markov chains resulted in better forecast performance than first order Markov forecasts.

Naveen et al. (2011) [13-17] attempted to apply second order markov chain concept to compute the optimal replacement age for a block of computer systems.

As the estimation of transition probabilities for bigger state space S =1,2,3,…,m is much time consuming one, Bruce A Criag et al (1998) [3] and Sutawanir *et al* (2008) [23] studied the spectral representation of transition probabilities and Zhenqing Li *et al* (2005) [27] tried computer aided program to estimate the high-order Transition Probability Matrix of the Markov Chain.

This paper discusses a mathematical model for group replacement of block of air conditioners with three intermediary states viz., minor repair state, semi-major repair state and major repair states between working state and breakdown state using first order and second order Markov Chains. Though it is difficult to identify the specific repairable intermediate state, to make the model simplistic the repairs are grouped as below.

Table – 1: Categorization of identified possible repairs in Air Conditioners

Minor repairs	Semi - Major repairs	Major repairs
Running capacitor problem Fan capacitor problem Temperature sensor problem Condenser fan motor problem Condenser blade problem Blower problem etc.	Capillary problem Gas leak problem Relay board problem Display board problem	Compressor Problem Evaporator problem Condenser coil problem etc.

The transition probabilities are estimated using first order and second order Markov Chains. Transition probabilities for future periods are estimated by Spectral Decomposition in first order Markov chain and by Moving Weighted Transition model for second order Markov chain. Also the influence of macroeconomic variables such as inflation and time value of money are considered to make the model yield better results.

The following sections deals with the theory of FOMC and SOMC, need for introducing WMTP method for predicting Transition Probabilities using SOMC, model development and a the results of a case study with a reference to a block of air conditioners.

II. PROBLEM FORMULATION

The objective of this study is to develop a mathematical model using Markov process for block replacement problem that considers the influence of inflation and time value of money on the optimal replacement policy. A special reference is given to the multi-state (breakdown) repairable system containing a block of similar items viz. Air Conditioners.

This model aims at finding the block replacement period for a group of air conditioners, that may consist of many intermediate repairable states like minor repair, semi-major repair and major repair etc. between functional and breakdown states that are more realistic. The fundamental objective of replacement is to direct the organization for maximizing its profit (or minimizing cost).Formulation of the problem is done in three steps:

i) Block replacement Model using first order Markov Chain.
ii) Block replacement Model using first order Markov chain considering macroeconomic variable i.e. Inflation.
iii) Block Replacement Model using higher order Markov chain considering macroeconomic variable i.e. Inflation.

III. MECHANICS OF MARKOV CHAIN

M*arkov process* is a stochastic or random process, which has property that the probability of transition from a given state to any future state depends on the present state and not on the manner in which it was reached.

3.1. First Order Markov Chain (FOMC)

The First Order Markov Chain (FOMC) assumes the probability of next state depends only on the immediately preceding state. Thus if $t_0 < t_1 < . . . < t_n$ represents the points on time scale then the family of random variables $\{X(t_n)\}$ whose state space S = 1,2, …,m is said to be a Markov process provided it holds the Markovian property:

$$P\{X(t_n) = X_n \mid X(t_{n-1}) = X_{n-1}, \ldots, X(t_0) = X_0\} = P\{X(t_n) = X_n \mid X(t_{n-1}) = X_{n-1}\} \ldots\ldots\ldots\ldots(1)$$

$$\text{for all } X(t_0), X(t_1), \ldots, X(t_n)$$

If the random process at time t_n is in the state x_n, the future state of the random process X_{n+1} at time t+1 depends only on the present state x_n and not on the past states $x_{n-1}, x_{n-2}, \ldots, x_0$. The simplest of the Markov Process is discrete and constant over time. A system is said to be discrete in time if it is examined at regular intervals, e.g. daily, monthly or yearly.

3.1.1. Transition Probability

The probability of a system moving from one state to another state or remaining in the same state during a given time period is called transition probability. Mathematically the probability:

$$P_{x(n-1),x(n)} = P\{X(t_n) = X_n \mid X(t_{n-1}) = X_{n-1}\} \ldots\ldots\ldots\ldots(2)$$

is called FOMC transition probability that represents the probability of the system moving from one state to another future state. The transition probabilities can be arranged in a matrix of size m x m and such a matrix can be called as one step Transition Probability Matrix (TPM), represented as shown in Figure 1.

$$P = \begin{bmatrix} P_{11} & P_{12} & \cdots & P_{1m} \\ P_{21} & P_{22} & \cdots & P_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ P_{m1} & P_{m2} & \cdots & P_{mm} \end{bmatrix}$$

Figure 1 One Step Transition Probability Matrix

Where 'm' represents the number of states. The matrix P is a square matrix of which each element is non-negative and sum of the elements in each row is unity i.e. $\sum_{j=1}^{m} P_{ij} = 1$; i= 1 to m and $0 \le P_{ij} \le 1$.

The initial estimates of P_{ij} can be computed as, $P_{ij} = \frac{N_{ij}}{N_i}$, (i, j = 1 to m) where N_{ij} is the raw data sample that refer the number of items or units or observations transitioned from the state i to state j. N_i is the raw data sample in state i.

3.1.2. Spectral Decomposition Method

As the estimation of high–order Markov chain transition probabilities for bigger state space S =1,2,3,…,m is much time consuming one, Bruce A Criag et al (1998) and Sutawanir *et al* (2008) studied the advantage of spectral representation of transition probabilities for multi state process and Zhenqing Li *et al* (2005) tried computer aided program to estimate the high-order Transition Probability Matrix of the Markov Chain. As several software for spectral decomposition are widely available [Sutawanir *et al* (2008)], this method provides flexibility for the computation of transition probabilities for multi state process.

Spectral Decomposition is based on eigen values. It is applicable to square matrix that will be decomposed into a product of three matrices, only one of which is diagonal matrix. As a result, the decomposition of a matrix into matrices composed of its eigen values and eigen vectors is called Eigen or Spectral decomposition.

An nxn matrix 'P' always has 'n' eigen values, which can be ordered (in more than one way) to form an n x n diagonal matrix D formed from the eigen values and a corresponding matrix, V, of non zero columns (eigen vectors) that satisfies the eigen value equation: PV = VD.

This gives the amazing decomposition of 'P' into a similarity transformation involving V and D as:

$$P = V D V^{-1} \quad \ldots\ldots(3)$$

Furthermore, squaring both sides of above equation gives:

$$P^2 = (V D V^{-1}) (V D V^{-1})$$
$$P^2 = V D (V^{-1}V) D V^{-1}$$
$$P^2 = V D^2 V^{-1} \quad \ldots\ldots(4)$$

Mathematically, Spectral Decomposition can be represented as $P^i = V D^i V^{-1}$ where i= 1 to n.

Therefore higher order Transition Probability Matrix (TPM) of five state Markov chain can be computed using the equation, $P^i = V D^i V^{-1}$ (where i= 1 to 5)……………………………(5)

3.2. Second Order Markov Chain (SOMC)

Second Order Markov Chain assumes that probability of next state depends on the probabilities of two immediately preceding states. Then we will have Second Order Markov Chain (SOMC) whose transition probabilities are:

$$P(X_{(n-2)}, X_{(n-1)}, X_{(n)}) = P\{X(t_n) = X_n \mid X(t_{n-1}) = X_{(n-1)}, X(t_{(n-2)}) = X_{(n-2)}\} \quad \ldots\ldots(6)$$

So as the model under study consists 5 states, the SOMC Transition Probability Matrix(TPM) (Andre Berchtold *et al* 2002) can be formulated as shown in Figure 2.

The size of the TPM will be m^l x m and the number of transition probabilities to be calculated in each TPM will be m^{l+1} . The Table–2 gives the number of transition probabilities for various combinations of order (l) and states (m).

Table–2 TPM sizes & No. of transition probabilities for various states and Markov Chain Order

No. of states (m)→	**2**		**3**		**4**		**5**	
Markov Chain Order (l)↓	**A**	**B**	**A**	**B**	**A**	**B**	**A**	**B**
1	2x2	4	3x3	9	4x4	16	5x5	25
2	4x2	8	9x3	27	16x4	64	25x5	125
3	8x2	16	27x3	81	64x4	256	125x5	625
4	16x2	32	81x3	243	256x4	1024	625x5	3125

5	32x2	64	243x3	729	1024x4	4096	3125x5	15625

Where, **Column A** = Size of the TPM, **Column B** = No. of Transition probabilities.

$$
TPM = P = \begin{pmatrix}
 & X_n & I & II & III & IV & V \\
X_{n-2} & X_{n-1} & & & & & \\
I & I & P_{111} & P_{112} & P_{113} & P_{114} & P_{115} \\
II & I & P_{211} & P_{212} & P_{213} & P_{214} & P_{215} \\
III & I & P_{311} & P_{312} & P_{313} & P_{314} & P_{315} \\
IV & I & P_{411} & P_{412} & P_{413} & P_{414} & P_{415} \\
V & I & P_{511} & P_{512} & P_{513} & P_{514} & P_{515} \\
I & II & \ldots & \ldots & \ldots & \ldots & \ldots \\
II & II & . & . & . & . & . \\
III & II & . & . & . & . & . \\
IV & II & . & . & . & . & . \\
V & II & . & . & . & . & . \\
I & III & . & . & . & . & . \\
II & III & . & . & . & . & . \\
III & III & . & . & . & . & . \\
IV & III & . & . & . & . & . \\
V & III & . & . & . & . & . \\
I & IV & . & . & . & . & . \\
II & IV & . & . & . & . & . \\
III & IV & . & . & . & . & . \\
IV & IV & . & . & . & . & . \\
V & IV & . & . & . & . & . \\
I & V & . & . & . & . & . \\
II & V & . & . & . & . & . \\
III & V & \ldots & \ldots & \ldots & \ldots & \ldots \\
IV & V & \ldots & \ldots & \ldots & \ldots & \ldots \\
V & V & P_{551} & P_{552} & P_{553} & P_{554} & P_{555}
\end{pmatrix}
$$

Figure 2 Transition Probability Matrix of Second Order Markov Chain with 5 states

3.3. Moving Weighted Transition (MWT) Probabilities

From Table-2 it is evident that for higher order Markov chain with more state space, the size of the TPM will be too large. Estimation of several hundreds of parameters is too difficult and time consuming as well. Also it will be difficult to analyze and make conclusions or decisions.

To address this, the Moving Weighted Transition (MWT) probabilities are introduced to estimate high order transition probabilities this is similar to Mixture Transition Distribution - MTD model (Raftery A E, 1985a) except the lag parameters are replaced by the weights. A detailed review on MTD is given by Berchtold and Raftery (2002). This makes the number of parameters in each TPM to be computed is far less. The size of the TPM is m x m only. Each element of the TPM is the probability for the occurrence of a particular state at time t given the probabilities of immediate previous *l* (= order of the Markov Chain) time periods. The effect of each lag is considered by assigning the weights.

For an *l*- order Markov Chain, in general, the probabilities can be estimated as:

$$P\{X(t_n) = X_n \mid X(t_{n-1}) = X_{n-1}, \ldots X(t_{n-l}) = X_{n-l}\} = \sum_{k=1}^{l} \omega_k (P_{ij})_k \qquad \ldots\ldots(7)$$

subject to $\sum_{k=1}^{l} \omega_k = 1$ and $\omega_k \geq 0$.

ω is the weight assigned to each previous probability in order to consider the effect of lag. In general more weightage is to be assigned to the immediate preceding transition probability and less weightage to the next preceding transition probability and so on. And P_{ij} are the transition probabilities of the corresponding m x m TPM.
The MWT probability for SOMC ($l = 2$) can be written as:

$$P\{X(t_n) = X_n \mid X(t_{n-1}) = X_{n-1}, X(t_{n-2}) = X_{n-2}\} = \sum_{k=1}^{2} \omega_k (P_{ij})_k = \omega_{n-1}(P_{ij})_{n-1} + \omega_{n-2}(P_{ij})_{n-2}$$

Where $\omega_{n-1} + \omega_{n-2} = 1 \; and \; \omega = 0.$ ……………………………………………………..(8)

As described by Kim *et al* (2009) as shown in the Figure 3 real high order Markov chain carries the combined influence of lags where as MWT model carries the independent influences of each lag.

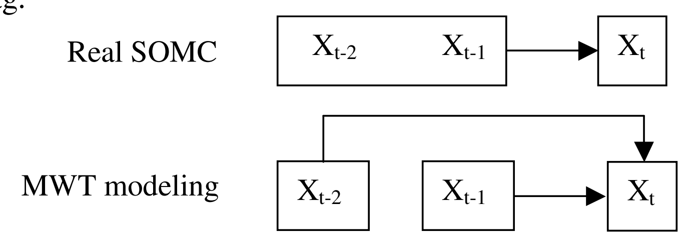

Figure 3 Influence of lags in real Higher Order Markov Chains & MWT Model

IV. MODEL DEVELOPMENT

4.1. Notations

N = Total Number of units in the System
C_1 = Individual Replacement Cost Per Unit
C_2 = Minor Repair Cost
C_3 = Semi-Major Repair Cost
C_4 = Major Repair Cost
C_5 = Group Replacement Cost
X_0^{I} = Proportion of units in working state initially
X_0^{II} = Proportion of units in minor repair state initially
X_0^{III} = Proportion of units in semi-major repair state initially
X_0^{IV} = Proportion of units in major repair state initially
X_0^{V} = Proportion of units in breakdown state initially
X_i^{I} = proportion of units in working state at the end of i^{th} time period
X_i^{II} = proportion of units in minor repair state at the end of i^{th} time period
X_i^{III} = proportion of units in semi major repair state at the end of i^{th} time period,
X_i^{IV} = proportion of units in major repair state at the end of i^{th} time period,
X_i^{V} = proportion of units in breakdown state at the end of i^{th} time period
P_{ij} = Probability of units switching from i^{th} state to j^{th} state in a period
TPM = Transition Probability Matrix
φ_t = Rate of Inflation during time 't'
r_n = Nominal Rate of Interest
r_t = Real Rate of Interest = $\frac{(r_n - \varphi_t)}{(1+\varphi_t)}$, from Fisherman's Relation…………………………(9)
Present Value Factor (PVF) = $\frac{1}{(1+r_t)}$………………………………………………………(10)
P_{ij} = Probability of units switching from i^{th} state to j^{th} state in a period
W(t) = Weighted average cost per period in group replacement policy,
AC(t) = average cost per period in group replacement policy

4.2. Problem Definition

In this paper, a group replacement model is developed for (N) items that fail completely on usage, considering three intermediate states i.e. minor repair, semi-major and major repair, between working and breakdown states by using First Order Markov Chain (FOMC). As the FOMC consists of five states, the Transition Probability Matrix (TPM) can be written as:

$$TPM = P = \begin{array}{c|ccccc} & I & II & III & IV & V \\ I & P_{11} & P_{12} & P_{13} & P_{14} & P_{15} \\ II & P_{21} & P_{22} & P_{23} & P_{24} & P_{25} \\ III & P_{31} & P_{32} & P_{33} & P_{34} & P_{35} \\ IV & P_{41} & P_{42} & P_{43} & P_{44} & P_{45} \\ V & P_{51} & P_{52} & P_{53} & P_{54} & P_{55} \end{array}$$

Figure 4 Transition Probability Matrix for FOMC with 5 states

Where I, II, III, IV and V represents Working, Minor repair, Semi-Major repair, Major repair and Break down States respectively.

Sum of the transition probabilities in each row is unity i.e. $\sum_{j=1}^{5} P_{ij} = 1$; i= 1 to 5

The initial estimates of P_{ij} can be computed as, $P_{ij} = \frac{N_{ij}}{N_i}$, (i, j = 1 to 5) ……………...........….(11)

Where,
N_{ij} is the number of Air Conditioners transitioned from the state i to state j.
N_i is the raw data sample in state i.
FOMC Transition probability, $P^i = V D^i V^{-1}$, where i = 1 to n, for 'n' future time periods can be computed using Spectral Decomposition Method [Sutawanir *et al* (2008)].

4.3. Calculation of number of items in each state

For both FOMC and SOMC, the proportion (X_i) of units during i^{th} period in various states i.e., the state probabilities of items in different states can be computed as follows.

$$[X_i^I \ X_i^{II} \ X_i^{III} \ X_i^{IV} \ X_i^V] = [X_0^I \ X_0^{II} \ X_0^{III} \ X_0^{IV} \ X_0^V] * P^i \text{ , where i = 1 to n……….(12)}$$

$$X_i = \text{(Probability of items in different states during initial period)} * (TPM)^i$$

In general, $X_i = X_0 P^i$ ……………………………………………………………………….(13)

The TPMs, $P^1, P^2, \ldots, P^i$ of future periods for FOMC can be calculated by spectral decomposition method. At the end of the first period, the state probabilities can be calculated from $X_1 = X_0 P$ $(\because X_i = X_0 P^i)$ ………………………………………………………. (14)

$$\Rightarrow [X_i^I \ X_i^{II} \ X_i^{III} \ X_i^{IV} \ X_i^V] = [X_0^I \ X_0^{II} \ X_0^{III} \ X_0^{IV} \ X_0^V] \begin{bmatrix} P_{11} & P_{12} & P_{13} & P_{14} & P_{15} \\ P_{21} & P_{22} & P_{23} & P_{24} & P_{25} \\ P_{31} & P_{32} & P_{33} & P_{34} & P_{35} \\ P_{41} & P_{42} & P_{43} & P_{44} & P_{45} \\ P_{51} & P_{52} & P_{53} & P_{54} & P_{55} \end{bmatrix}$$

Therefore, Probability of items in functional state, at the end of the first period

$$X_1^I = X_0^I P_{11} + X_0^{II} P_{21} + X_0^{III} P_{31} + X_0^{IV} P_{41} + X_0^V P_{51} \text{ ……………………….(15)}$$

Probability of items in minor repair state, at the end of the first period

$$X_1^{II} = X_0^I P_{12} + X_0^{II} P_{22} + X_0^{III} P_{32} + X_0^{IV} P_{42} + X_0^V P_{52} \text{ …………………...…(16)}$$

Probability of items in semi-major repair state, at the end of the first period

$$X_1^{III} = X_0^I P_{13} + X_0^{II} P_{23} + X_0^{III} P_{33} + X_0^{IV} P_{43} + X_0^V P_{53} \text{ ……………….…….(17)}$$

Probability of items in major repair state, at the end of the first period

$$X_1^{IV} = X_0^{I}P_{14} + X_0^{II}P_{24} + X_0^{III}P_{34} + X_0^{IV}P_{44} + X_0^{V}P_{54} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(18)$$

Probability of items in irreparable state, at the end of the first period

$$X_1^{V} = X_0^{I}P_{15} + X_0^{II}P_{25} + X_0^{III}P_{35} + X_0^{IV}P_{45} + X_0^{V}P_{55} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(19)$$

Similarly, the probabilities of items falling in different states in future time periods (i= 1 to n) are to be calculated by using the equation, $X_n = X_0 P^n$ ……………………………………(20)

The TPMs (P^i) for future 'n' time periods i = 1,2,…, n are calculated by using Spectral Decomposition method. Using these state probabilities the number of individual replacements (α_i), minor repairs (β_i), semi-major repairs (γ_i) and major repairs (δ_i) in future time periods can be calculated as shown in Table-3.

Table 3 No. of various repairs at the end of each period

Time period	No. of individual replacements	No. of minor repairs
1st	$\alpha_1 = NX_1^V$	$\beta_1 = NX_1^{II}$
2nd	$\alpha_2 = NX_2^V + \alpha_1 X_1^V$	$\beta_2 = NX_2^{II} + \beta_1 X_1^{II}$
3rd	$\alpha_3 = NX_3^V + \alpha_1 X_2^V + \alpha_2 X_1^V$	$\beta_3 = NX_3^{II} + \beta_1 X_2^{II} + \beta_2 X_1^{II}$
4th	$\alpha_4 = NX_4^V + \alpha_1 X_3^V + \alpha_2 X_2^V + \alpha_3 X_1^V$	$\beta_4 = NX_4^{II} + \beta_1 X_3^{II} + \beta_2 X_2^{II} + \beta_3 X_1^{II}$
5th	$\alpha_5 = NX_5^V + \alpha_1 X_4^V + \alpha_2 X_3^V + \alpha_3 X_2^V + \alpha_4 X_1^V$	$\beta_5 = NX_5^{II} + \beta_1 X_4^{II} + \beta_2 X_3^{II} + \beta_3 X_2^{II} + \beta_4 X_1^{II}$
. . . S o on	. . . S o on	. . . S o on

Table 3 *(Continued)* No. of various repairs at the end of each period

Time period	No. of semi-major repairs	No. of major repairs
1st	$\gamma_1 = NX_1^{III}$	$\delta_1 = NX_1^{IV}$
2nd	$\gamma_2 = NX_2^{III} + \gamma_1 X_1^{III}$	$\delta_2 = NX_2^{IV} + \delta_1 X_1^{IV}$
3rd	$\gamma_3 = NX_3^{III} + \gamma_1 X_2^{III} + \gamma_2 X_1^{III}$	$\delta_3 = NX_3^{IV} + \delta_1 X_2^{IV} + \delta_2 X_1^{IV}$
4th	$\gamma_4 = NX_4^{III} + \gamma_1 X_3^{III} + \gamma_2 X_2^{III} + \gamma_3 X_1^{III}$	$\delta_4 = NX_4^{IV} + \delta_1 X_3^{IV} + \delta_2 X_2^{IV} + \delta_3 X_1^{IV}$
5th	$\gamma_5 = NX_5^{III} + \gamma_1 X_4^{III} + \gamma_2 X_3^{III} + \gamma_3 X_2^{III} + \gamma_4 X_1^{III}$	$\delta_5 = NX_5^{IV} + \delta_1 X_4^{IV} + \delta_2 X_3^{IV} + \delta_3 X_2^{IV} + \delta_4 X_1^{IV}$
. .. S o on	. . .S o on	. . S o on

4.4. Inflation Prediction

Inflation is predicted using regression model with trigonometric function and the influence of Inflation on the prices of air conditioners in India from the year 1999 onwards over a period of time is studied, forecasted and compared with actual values for the known periods by employing various forecasting techniques to identify the underlying model that best fits the time series data. Subsequently the inflation is predicted for *Air Conditioners* for the future time periods by the developed Regression model with trigonometric function, which yielded relatively minimal errors.

A sinusoidal trigonometric function is used in the regression model to accommodate cyclical fluctuations of inflation. For this the following mathematical equation is considered.

$$\phi = a + bt + c \sin(t\pi + \pi/4) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(21)$$

To find the constants a, b & c the following set of equations are used.

$$\sum\phi = na + b\sum t + c\sum\sin(t\pi + \pi/4) \quad \text{.....(22)}$$

$$\sum(\phi t) = a\sum t + b\sum t^2 + c\sum[t\sin(t\pi + \pi/4)] \quad \text{.....(23)}$$

$$\sum(\phi t^2) = a\sum t^2 + b\sum t^3 + c\sum[t^2\sin(t\pi + \pi/4)] \quad \text{.....(24)}$$

where ϕ is the inflation, t is time period, n is the number of time periods and a, b & c are the coefficients.

4.5. Influence of Inflation and Time Value of Money

Conventional models are available to make the replacement decisions considering the value of money. Here the Net Present Worth criterion based on the nominal interest rate (r_n) does not reflect the real value of money. Real interest rates (r_t) are computed using Fisherman's relation. When the present worth factors are computed and multiplied with future money, it gives purchasing power of money.

Present worth factor= pwf= $\nu = \frac{1}{1+r_t}$ ....(25)

Real rate of interest = $r_t = \frac{r_n - \varphi_t}{1 + \varphi_t}$, from Fisherman's relation..........(26)

To get more realistic results, the forecasted values of Inflation to get the real interest rates, using Fisherman's relation, are used. The Inflation values based on WPI for *air conditioners* are predicted by using a regression forecasting model using a sinusoidal trigonometric function.

4.6. Computation of Total Cost

Total cost (TC) for n time periods, without the influence of Inflation:
TC = Group replacement cost + Individual replacement cost + Minor repair cost + Semi – Major repair cost + Major repair cost

$$TC = NC_5 + C_1\sum_{i=1}^{n}\alpha_i + C_2\sum_{i=1}^{n}\beta_i + C_3\sum_{i=1}^{n}\gamma_i + C_4\sum_{i=1}^{n}\delta_i \quad \text{.....(27)}$$

$$\text{Average cost per period} = \frac{\text{Total cost for 'n' periods}}{\text{number of periods}}$$

$$\text{Weighted average cost, } W(t) = TC/n$$

Total cost (TC) for n time periods, with the influence of inflation.

$$TC(n) = C_1\left[\alpha_1 + \alpha_2\nu + \alpha_3\nu^2 + \ldots + \alpha_n\nu^{n-1}\right] + C_2\left[\beta_1 + \beta_2\nu + \beta_3\nu^2 + \ldots + \beta_n\nu^{n-1}\right]$$
$$+ C_3\left[\gamma_1 + \gamma_2\nu + \gamma_3\nu^2 + \ldots + \gamma_n\nu^{n-1}\right] + C_4\left[\delta_1 + \delta_2\nu + \delta_3\nu^2 + \ldots + \delta_n\nu^{n-1}\right] + NC_5\nu^{n-1}$$

$$TC(n) = C_1\sum(\alpha_n\nu^{n-1}) + C_2\sum(\beta_n\nu^{n-1}) + C_3\sum(\gamma_n\nu^{n-1}) + C_4\sum(\delta_n\nu^{n-1}) + NC_5\nu^{n-1} \quad \text{.....(28)}$$

Weighted average cost, $W(t) = TC/\sum\nu^{n-1}$ ....(29)

Policy: 'n' is optimal when the weighted average cost per period is minimum i.e. average cost per period should be minimum in n^{th} period, to block replace in n^{th} period.

V. CASE STUDY

In the present paper, a study on a block of 100 air conditioners in a software development centre have been considered and the cost data for various types of repairs based on the information given by the air conditioners service engineers is assumed as given below.

N = Total number of Air Conditioners in the system	=	100
C_1 = Individual replacement cost per unit	=	Rs.23000
C_2 = Minor repair cost	=	Rs.4500
C_3 = Semi-major repair cost	=	Rs.8000

C_4 = Major repair cost = Rs.15500
C_5 = Group replacement cost = Rs.21000
r_n = Nominal rate of Interest = 20%

$$X_0 = \left[X_0^{I} \quad X_0^{II} \quad X_0^{III} \quad X_0^{IV} \quad X_0^{V}\right] = \left[0.850 \quad 0.090 \quad 0.030 \quad 0.020 \quad 0.010\right]$$

$$P^1 = TPM = \begin{bmatrix} 0.6588 & 0.1765 & 0.0471 & 0.1059 & 0.0118 \\ 0.6667 & 0.1111 & 0.1111 & 0.1111 & 0.0000 \\ 0.3333 & 0.3333 & 0.3333 & 0.0000 & 0.0000 \\ 0.5000 & 0.5000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 \end{bmatrix}$$

Regression model with trigonometric function for predicting inflation for a time period t is

$$F = -4.50352 + 0.30617T + 7.55993 \, \text{Sin}\,(T\pi + \pi/4) \qquad \ldots\ldots(30)$$

The average cost per year using First Order Markov Chain (FOMC) and Second Order Markov Chain (SOMC) are shown in the Table-4.

Table 4: Comparison of Average Cost in lakhs per year for both FOMC & SOMC

Time period(n)	**FOMC**		**SOMC**
	Without inflation	with inflation	with inflation
1	24.30	24.30	24.30
2	14.05	12.50	12.50
3	10.83	9.77	9.71
4	9.37	**7.01***	**6.91***
5	8.63	7.31	7.15
6	8.26		
7	**8.12***		
8	8.13		

* Minimum Cost giving Optimum period for group replacement

When the influence of inflation is not considered, First Order Markov Chain (FOMC) model resulted in the replacement age as 7 years.
When the influence of predicted inflation and Net Worth of the money is considered, both First Order Markov Chain (FOMC) and Second Order Markov Chain (SOMC) has resulted in the early replacement of block of Air Conditioners at the age of 4 years. Therefore for the block of air conditioners considered for study in this work, the optimal replacement time is at the end of 4^{th} year.

VI. CONCLUSIONS

A discrete time Markov Chain based mathematical model for the replacement of block of items has been developed and a case study is made with a reference to a block of air conditioners. Five discrete states viz. working condition, minor repair, semi-major repair, major repair and breakdown state of an air conditioning system are considered and the results are computed using First Order Markov Chain (FOMC) and Second Order Markov Chain (SOMC) with and without the influence of Inflation. FOMC has resulted in optimal replacement age of 7 years without considering inflation and money value, where as considering inflation and time value of money it was 4 years for both First Order and Second Order Markov Chains. Since, FOMC and SOMC resulted in the same replacement period of 4 years, therefore for the block of air conditioners considered in the study, the optimal replacement period is four years. As the influence of macroeconomic variables viz. inflation and time value of money on replacement model are considered, the model is more realistic.

VII. SCOPE OF FUTURE WORK

In this paper, only three intermediate states (minor, semi-major and major repair) between working and break down states are considered i.e. only five general states of items are considered. Practically, there may be varieties of failures and for each failure different repair costs can be attached. It is difficult to consider a particular failure as minor or semi-major or major repair. Therefore, few more states may be proposed so that a type of failure will be very close to a particular state. Also, in this paper, nominal interest rate is assumed to be constant. Usually nominal interest rates will be revised based on the changes in macro economic variables in the free market economic conditions. Therefore variable rate of nominal interest is proposed as one of the inputs for further study of replacement decision.

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